Sample Analysis: Torsion of Composite “S” Section

Consider the composite section shown in Figure 1.

Figure 1. Composite Cross-Section (all dimensions are in inches)
For this cross-section, let

- \( G_1 = 15 \times 10^6 \text{ psi} \)
- \( G_2 = 45 \times 10^6 \text{ psi} \)
- Angle of twist per unit length (\( \Theta \)) = 1.40 \times 10^{-3} \text{ radians/in.} 

It follows that the “source terms” associated with the two materials are thus:

- \( S_1 = 2G_1\Theta = 2(15 \times 10^6)(1.40 \times 10^{-3}) = 4.20 \times 10^4 \text{ lb/in}^3 \)
- \( S_2 = 2G_2\Theta = 2(45 \times 10^6)(1.40 \times 10^{-3}) = 1.26 \times 10^5 \text{ lb/in}^3 \)

We seek to estimate the maximum value of the Prandtl stress function \( \Phi(x, y) \) and the maximum values and locations of the shear stresses \( \sigma_{13} \) and \( \sigma_{23} \), where

\[
\sigma_{13} = \frac{\partial \Phi}{\partial y}, \quad \sigma_{23} = -\frac{\partial \Phi}{\partial x}
\]

The first mesh considered is shown in Figure 2. While it is rather coarse, it is not the coarsest mesh possible. Note that the missing node option has been used in discretizing the domain.
Figure 2. 32-Element Mesh of 4-Node (QS4) Quadrilaterals

The UD_scalar data file associated with this mesh is shown below.
anal tit "torsion of an 'S' cross-section"
anal tit "comprised of two materials"
anal tit "via Prandtl's approach"
anal tit "using 4-node quadrilateral elements"
anal tit "first mesh: 32 elements"

analysis action analyze
analysis temporal static

! echo warn off
!
dim max scalar_1 2
dim max nodes 65
dim max qs4 32
!
finish settings
!
scalar source number 1 source_1 4.200E+04
!
scalar source number 2 source_1 1.260E+05
!
scalar conduc constant number 1 &
  desc "this is material #1" k11 1.0 k22 1.0
!
scalar conduc constant number 2 &
  desc "this is material #2" k11 1.0 k22 1.0
!

nodes line number 1
nodes line number 3  x1 5.0  inc 1
nodes line number 13  x1 5.0  x2 5.0  inc 5
nodes line number 11  x2 5.0  inc -1
nodes line number -1  inc -5
!

nodes line number -3
nodes line number 5  x1 11.0  inc 1
nodes line number 15  x1 11.0  x2 5.0  inc 5
nodes line number -13  inc -1
nodes line number -3  inc -5
!

nodes line number -13
nodes line number -15  inc 1
nodes line number 50  x1 11.0  x2 25.0  inc 5
nodes line number 48  x1 5.0  x2 25.0  inc -1
nodes line number -13  inc -5
!

nodes line number -48
nodes line number 53  x1 20.0  x2 25.0  inc 1
nodes line number 65  x1 20.0  x2 30.0  inc 6
nodes line number 60  x1 5.0  x2 30.0  inc -1
nodes line number -48  inc -6
!

element scalar typ "qs4" nodes 1 2 7 6 scalar 1 &
  _add 1 _incr 1 2_add 1 2_incr 5
element scalar typ "qs4" nodes 3 4 9 8 scalar 1 &
  2_add 8 2_incr 5
element scalar typ "qs4" nodes 48 49 55 54 scalar 1 &
  2_add 1 2_incr 6
The resulting *UD_scalar* output file generated using the above data is shown below.
torsion of an 'S' cross-section
comprised of two materials
via Prandtl's approach
using 4-node quadrilateral elements
first mesh: 32 elements

Largest NODE number which can be used in the mesh = 65
Max. no. of CONSTANT scalar conductivity idealizations = 2
Max. no. of 4-node quad. "scalar" (QS4) elements = 32

-- analysis with SCALAR primary dependent variables shall be performed

-- TWO-DIMENSIONAL solution domain assumed
(PLANE STRESS idealization)

-- solver type used : SKYLINE

-- storage type : SYMMETRIC

-- "Isoparametric" mesh generation scheme used
-> LINEAR analysis

======================================================================
= SCALAR MATERIAL PARAMETERS =
======================================================================

-> idealization no.: 1

 type: constant scalar conductivity coefficients
 info.: this is material #1

"Conductivities" (material parameters):

\[
\begin{align*}
  k_{11} &= 1.000E+00 & k_{12} &= 0.000E+00 & k_{13} &= 0.000E+00 \\
  k_{22} &= 1.000E+00 & k_{23} &= 0.000E+00 & k_{33} &= 1.000E+00 \\
\end{align*}
\]

source term $S_1 = 4.200E+04$
source term $S_2 = 0.000E+00$

-> idealization no.: 2

 type: constant scalar conductivity coefficients
 info.: this is material #2

"Conductivities" (material parameters):

\[
\begin{align*}
  k_{11} &= 1.000E+00 & k_{12} &= 0.000E+00 & k_{13} &= 0.000E+00 \\
  k_{22} &= 1.000E+00 & k_{23} &= 0.000E+00 & k_{33} &= 1.000E+00 \\
\end{align*}
\]

source term $S_1 = 1.260E+05$
source term $S_2 = 0.000E+00$

======================================================================
= NODAL COORDINATES =
======================================================================

<table>
<thead>
<tr>
<th>node</th>
<th>x1</th>
<th>x2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-7.523E-37</td>
<td>-7.523E-37</td>
</tr>
<tr>
<td>2</td>
<td>2.500E+00</td>
<td>7.523E-37</td>
</tr>
<tr>
<td>3</td>
<td>5.000E+00</td>
<td>7.521E-37</td>
</tr>
<tr>
<td>4</td>
<td>8.000E+00</td>
<td>7.521E-37</td>
</tr>
<tr>
<td>5</td>
<td>1.100E+01</td>
<td>7.521E-37</td>
</tr>
<tr>
<td>6</td>
<td>-1.505E-36</td>
<td>2.500E+00</td>
</tr>
<tr>
<td>7</td>
<td>2.500E+00</td>
<td>2.500E+00</td>
</tr>
<tr>
<td>8</td>
<td>5.000E+00</td>
<td>2.500E+00</td>
</tr>
<tr>
<td>9</td>
<td>8.000E+00</td>
<td>2.500E+00</td>
</tr>
<tr>
<td>10</td>
<td>1.100E+01</td>
<td>2.500E+00</td>
</tr>
<tr>
<td>11</td>
<td>-3.762E-37</td>
<td>5.000E+00</td>
</tr>
<tr>
<td>12</td>
<td>2.500E+00</td>
<td>5.000E+00</td>
</tr>
<tr>
<td>13</td>
<td>5.000E+00</td>
<td>5.000E+00</td>
</tr>
<tr>
<td>14</td>
<td>8.000E+00</td>
<td>5.000E+00</td>
</tr>
</tbody>
</table>
node :  15  x1 =  1.100E+01  x2 =  5.000E+00
node :  18  x1 =  5.000E+00  x2 =  7.857E+00
node :  19  x1 =  8.000E+00  x2 =  1.071E+01
node :  20  x1 =  1.100E+01  x2 =  1.071E+01
node :  23  x1 =  8.000E+00  x2 =  1.357E+01
node :  24  x1 =  1.100E+01  x2 =  1.357E+01
node :  25  x1 =  1.100E+01  x2 =  1.643E+01
node :  26  x1 =  1.100E+01  x2 =  1.929E+01
node :  27  x1 =  1.100E+01  x2 =  2.214E+01
node :  28  x1 =  1.100E+01  x2 =  2.500E+01
node :  29  x1 =  1.100E+01  x2 =  2.750E+01
node :  30  x1 =  1.100E+01  x2 =  3.000E+01
node :  31  x1 =  1.100E+01  x2 =  3.250E+01
node :  32  x1 =  1.100E+01  x2 =  3.500E+01

element information

---> number :  1  (type : QS4 )  (kind : SCALAR       )

      nodes :   1   2    7   6

descriptive info. :
integration rule : two-point Gauss-Legendre
scalar conductivity no. =   1
thickness =  1.000E+00

d---> number :  2  (type : QS4 )  (kind : SCALAR       )

      nodes :   2   3    8   7

descriptive info. :
integration rule : two-point Gauss-Legendre
scalar conductivity no. = 1
thickness = 1.000E+00

---> number : 3 (type : QS4 ) (kind : SCALAR )
~~~~~~ nodes : 6 7 12 11

descriptive info. :
integration rule : two-point Gauss-Legendre
scalar conductivity no. = 1
thickness = 1.000E+00

---> number : 4 (type : QS4 ) (kind : SCALAR )
~~~~~~ nodes : 7 8 13 12

descriptive info. :
integration rule : two-point Gauss-Legendre
scalar conductivity no. = 1
thickness = 1.000E+00

---> number : 5 (type : QS4 ) (kind : SCALAR )
~~~~~~ nodes : 3 4 9 8

descriptive info. :
integration rule : two-point Gauss-Legendre
scalar conductivity no. = 1
thickness = 1.000E+00

---> number : 6 (type : QS4 ) (kind : SCALAR )
~~~~~~ nodes : 8 9 14 13

descriptive info. :
integration rule : two-point Gauss-Legendre
scalar conductivity no. = 1
thickness = 1.000E+00

---> number : 7 (type : QS4 ) (kind : SCALAR )
~~~~~~ nodes : 13 14 19 18

descriptive info. :
integration rule : two-point Gauss-Legendre
scalar conductivity no. = 1
thickness = 1.000E+00

---> number : 8 (type : QS4 ) (kind : SCALAR )
<table>
<thead>
<tr>
<th>Number</th>
<th>Type</th>
<th>Kind</th>
<th>Nodes</th>
<th>Descriptive Info.</th>
</tr>
</thead>
</table>
| 9      | QS4   | SCALAR     | 23, 24, 29, 28 | integration rule: two-point Gauss-Legendre  
|        |       |            |         | scalar conductivity no. = 1                                                        
|        |       |            |         | thickness = 1.000E+00                                                             |
| 10     | QS4   | SCALAR     | 28, 29, 34, 33 | integration rule: two-point Gauss-Legendre  
|        |       |            |         | scalar conductivity no. = 1                                                        
|        |       |            |         | thickness = 1.000E+00                                                             |
| 11     | QS4   | SCALAR     | 33, 34, 39, 38 | integration rule: two-point Gauss-Legendre  
|        |       |            |         | scalar conductivity no. = 1                                                        
|        |       |            |         | thickness = 1.000E+00                                                             |
| 12     | QS4   | SCALAR     | 38, 39, 44, 43 | integration rule: two-point Gauss-Legendre  
|        |       |            |         | scalar conductivity no. = 1                                                        
|        |       |            |         | thickness = 1.000E+00                                                             |
| 13     | QS4   | SCALAR     | 43, 44, 49, 48 | integration rule: two-point Gauss-Legendre  
|        |       |            |         | scalar conductivity no. = 1                                                        
|        |       |            |         | thickness = 1.000E+00                                                             |
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-- number: 14 (type: QS4) (kind: SCALAR)

nodes: 48 49 55 54

descriptive info.:
integration rule: two-point Gauss-Legendre
scalar conductivity no. = 1
thickness = 1.000E+00

-- number: 15 (type: QS4) (kind: SCALAR)

nodes: 54 55 61 60

descriptive info.:
integration rule: two-point Gauss-Legendre
scalar conductivity no. = 1
thickness = 1.000E+00

-- number: 16 (type: QS4) (kind: SCALAR)

nodes: 4 5 10 9

descriptive info.:
integration rule: two-point Gauss-Legendre
scalar conductivity no. = 2
thickness = 1.000E+00

-- number: 17 (type: QS4) (kind: SCALAR)

nodes: 9 10 15 14

descriptive info.:
integration rule: two-point Gauss-Legendre
scalar conductivity no. = 2
thickness = 1.000E+00

-- number: 18 (type: QS4) (kind: SCALAR)

nodes: 14 15 20 19

descriptive info.:
integration rule: two-point Gauss-Legendre
scalar conductivity no. = 2
thickness = 1.000E+00

-- number: 19 (type: QS4) (kind: SCALAR)

nodes: 19 20 25 24

descriptive info.:
integration rule: two-point Gauss-Legendre
scalar conductivity no. = 2
thickness = 1.000E+00

---> number : 20  (type : QS4 )  (kind : SCALAR )
     ~~~~~ nodes :  24  25  30  29

descriptive info. :
integration rule : two-point Gauss-Legendre
scalar conductivity no. = 2
thickness = 1.000E+00

---> number : 21  (type : QS4 )  (kind : SCALAR )
     ~~~~~ nodes :  29  30  35  34

descriptive info. :
integration rule : two-point Gauss-Legendre
scalar conductivity no. = 2
thickness = 1.000E+00

---> number : 22  (type : QS4 )  (kind : SCALAR )
     ~~~~~ nodes :  34  35  40  39

descriptive info. :
integration rule : two-point Gauss-Legendre
scalar conductivity no. = 2
thickness = 1.000E+00

---> number : 23  (type : QS4 )  (kind : SCALAR )
     ~~~~~ nodes :  39  40  45  44

descriptive info. :
integration rule : two-point Gauss-Legendre
scalar conductivity no. = 2
thickness = 1.000E+00

---> number : 24  (type : QS4 )  (kind : SCALAR )
     ~~~~~ nodes :  44  45  50  49

descriptive info. :
integration rule : two-point Gauss-Legendre
scalar conductivity no. = 2
thickness = 1.000E+00

---> number : 25  (type : QS4 )  (kind : SCALAR )
     ~~~~~ nodes :  49  50  56  55
descriptive info. :
integration rule : two-point Gauss-Legendre
scalar conductivity no. = 2
thickness = 1.000E+00

---> number : 26 (type: QS4) (kind: SCALAR)

nodes : 50 51 57 56

descriptive info. :
integration rule : two-point Gauss-Legendre
scalar conductivity no. = 2
thickness = 1.000E+00

---> number : 27 (type: QS4) (kind: SCALAR)

nodes : 51 52 58 57

descriptive info. :
integration rule : two-point Gauss-Legendre
scalar conductivity no. = 2
thickness = 1.000E+00

---> number : 28 (type: QS4) (kind: SCALAR)

nodes : 52 53 59 58

descriptive info. :
integration rule : two-point Gauss-Legendre
scalar conductivity no. = 2
thickness = 1.000E+00

---> number : 29 (type: QS4) (kind: SCALAR)

nodes : 55 56 62 61

descriptive info. :
integration rule : two-point Gauss-Legendre
scalar conductivity no. = 2
thickness = 1.000E+00

---> number : 30 (type: QS4) (kind: SCALAR)

nodes : 56 57 63 62

descriptive info. :
integration rule : two-point Gauss-Legendre
scalar conductivity no. = 2
thickness = 1.000E+00
--- number :  31   (type : QS4   )   (kind : SCALAR       )

      nodes :   57  58  64  63

  descriptive info. :
  integration rule : two-point Gauss-Legendre
  scalar conductivity no. =   2
  thickness =  1.000E+00

---

--- number :  32   (type : QS4   )   (kind : SCALAR       )

      nodes :   58  59  65  64

  descriptive info. :
  integration rule : two-point Gauss-Legendre
  scalar conductivity no. =   2
  thickness =  1.000E+00

--- NODE POINT SPECIFICATIONS =

<table>
<thead>
<tr>
<th>Node Number</th>
<th>Coordinates</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(x1 = -7.523E-37, x2 = -7.523E-37)</td>
<td>phi = 0.000E+00</td>
</tr>
<tr>
<td>2</td>
<td>(x1 = 2.500E+00, x2 = 7.523E-37)</td>
<td>phi = 0.000E+00</td>
</tr>
<tr>
<td>3</td>
<td>(x1 = 5.000E+00, x2 = 7.521E-37)</td>
<td>phi = 0.000E+00</td>
</tr>
<tr>
<td>4</td>
<td>(x1 = 8.000E+00, x2 = 7.521E-37)</td>
<td>phi = 0.000E+00</td>
</tr>
<tr>
<td>5</td>
<td>(x1 = 1.100E+01, x2 = 7.521E-37)</td>
<td>phi = 0.000E+00</td>
</tr>
<tr>
<td>6</td>
<td>(x1 = -1.505E-36, x2 = 2.500E+00)</td>
<td>phi = 0.000E+00</td>
</tr>
<tr>
<td>10</td>
<td>(x1 = 1.100E+01, x2 = 2.500E+00)</td>
<td>phi = 0.000E+00</td>
</tr>
<tr>
<td>11</td>
<td>(x1 = -3.762E-37, x2 = 5.000E+00)</td>
<td>phi = 0.000E+00</td>
</tr>
<tr>
<td>12</td>
<td>(x1 = 2.500E+00, x2 = 5.000E+00)</td>
<td>phi = 0.000E+00</td>
</tr>
</tbody>
</table>
phi (essential specification) = 0.000E+00

13 : ( x1 = 5.000E+00, x2 = 5.000E+00 )
phi (essential specification) = 0.000E+00

15 : ( x1 = 1.100E+01, x2 = 5.000E+00 )
phi (essential specification) = 0.000E+00

18 : ( x1 = 5.000E+00, x2 = 7.857E+00 )
phi (essential specification) = 0.000E+00

20 : ( x1 = 1.100E+01, x2 = 7.857E+00 )
phi (essential specification) = 0.000E+00

23 : ( x1 = 5.000E+00, x2 = 1.071E+01 )
phi (essential specification) = 0.000E+00

25 : ( x1 = 1.100E+01, x2 = 1.071E+01 )
phi (essential specification) = 0.000E+00

28 : ( x1 = 5.000E+00, x2 = 1.357E+01 )
phi (essential specification) = 0.000E+00

30 : ( x1 = 1.100E+01, x2 = 1.357E+01 )
phi (essential specification) = 0.000E+00

33 : ( x1 = 5.000E+00, x2 = 1.643E+01 )
phi (essential specification) = 0.000E+00

35 : ( x1 = 1.100E+01, x2 = 1.643E+01 )
phi (essential specification) = 0.000E+00

38 : ( x1 = 5.000E+00, x2 = 1.929E+01 )
phi (essential specification) = 0.000E+00

40 : ( x1 = 1.100E+01, x2 = 1.929E+01 )
phi (essential specification) = 0.000E+00

43 : ( x1 = 5.000E+00, x2 = 2.214E+01 )
phi (essential specification) = 0.000E+00

45 : ( x1 = 1.100E+01, x2 = 2.214E+01 )
phi (essential specification) = 0.000E+00

48 : ( x1 = 5.000E+00, x2 = 2.500E+01 )
phi (essential specification) = 0.000E+00

50 : ( x1 = 1.100E+01, x2 = 2.500E+01 )
phi (essential specification) = 0.000E+00

51 : ( x1 = 1.400E+01, x2 = 2.500E+01 )
phi (essential specification) = 0.000E+00

52 : ( x1 = 1.700E+01, x2 = 2.500E+01 )
phi (essential specification) = 0.000E+00

53 : ( x1 = 2.000E+01, x2 = 2.500E+01 )
phi (essential specification) = 0.000E+00

54: ( x1 = 5.000E+00, x2 = 2.750E+01 )
phi (essential specification) = 0.000E+00

59: ( x1 = 2.000E+01, x2 = 2.750E+01 )
phi (essential specification) = 0.000E+00

60: ( x1 = 5.000E+00, x2 = 3.000E+01 )
phi (essential specification) = 0.000E+00

61: ( x1 = 8.000E+00, x2 = 3.000E+01 )
phi (essential specification) = 0.000E+00

62: ( x1 = 1.100E+01, x2 = 3.000E+01 )
phi (essential specification) = 0.000E+00

63: ( x1 = 1.400E+01, x2 = 3.000E+01 )
phi (essential specification) = 0.000E+00

64: ( x1 = 1.700E+01, x2 = 3.000E+01 )
phi (essential specification) = 0.000E+00

65: ( x1 = 2.000E+01, x2 = 3.000E+01 )
phi (essential specification) = 0.000E+00

~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
end of mathematical model data
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

======================================================================
=                  E L E M E N T     F L U X E S                     =
======================

-->
-> element 1 ( type = QS4 ) : [

.................
@ (x1 = 1.250E+00, x2 = 1.250E+00) : flux_1 = -2.444E+04 ; flux_2 = -
2.444E+04

-->
-> element 2 ( type = QS4 ) : [

.................
@ (x1 = 1.250E+00, x2 = 1.250E+00) : flux_1 = -2.444E+04 ; flux_2 = -
6.248E+04

-->
-> element 3 ( type = QS4 ) : [

.................
@ (x1 = 1.250E+00, x2 = 3.750E+00) : flux_1 = -2.444E+04 ; flux_2 =
2.444E+04

-->
-> element 4 ( type = QS4 ) : [

]
\[
\theta(x_1 = 3.750E+00, x_2 = 3.750E+00) : \text{flux}_1 = -1.360E+04 ; \text{flux}_2 = 6.248E+04
\]

---> element 5 ( type = QS4 ) : [ ]

\[
\theta(x_1 = 6.500E+00, x_2 = 1.250E+00) : \text{flux}_1 = -2.139E+04 ; \text{flux}_2 = -1.017E+05
\]

---> element 6 ( type = QS4 ) : [ ]

\[
\theta(x_1 = 6.500E+00, x_2 = 3.750E+00) : \text{flux}_1 = -8.650E+04 ; \text{flux}_2 = 2.361E+04
\]

---> element 7 ( type = QS4 ) : [ ]

\[
\theta(x_1 = 6.500E+00, x_2 = 6.429E+00) : \text{flux}_1 = -1.284E+05 ; \text{flux}_2 = 1.894E+03
\]

---> element 8 ( type = QS4 ) : [ ]

\[
\theta(x_1 = 6.500E+00, x_2 = 9.286E+00) : \text{flux}_1 = -1.264E+05 ; \text{flux}_2 = -1.732E+01
\]

---> element 9 ( type = QS4 ) : [ ]

\[
\theta(x_1 = 6.500E+00, x_2 = 1.214E+01) : \text{flux}_1 = -1.261E+05 ; \text{flux}_2 = 1.748E+04
\]

---> element 10 ( type = QS4 ) : [ ]

\[
\theta(x_1 = 6.500E+00, x_2 = 1.500E+01) : \text{flux}_1 = -1.260E+05 ; \text{flux}_2 = -1.086E+03
\]

---> element 11 ( type = QS4 ) : [ ]

\[
\theta(x_1 = 6.500E+00, x_2 = 1.786E+01) : \text{flux}_1 = -1.274E+05 ; \text{flux}_2 = -7.418E+03
\]

---> element 12 ( type = QS4 ) : [ ]

\[
\theta(x_1 = 6.500E+00, x_2 = 2.071E+01) : \text{flux}_1 = -1.355E+05 ; \text{flux}_2 = -1.086E+03
\]

---> element 13 ( type = QS4 ) : [ ]

\[
\theta(x_1 = 6.500E+00, x_2 = 2.357E+01) : \text{flux}_1 = -1.280E+05 ; \text{flux}_2 = 1.748E+04
\]
element

15 ( type = QS4 ) :

\[ @ (x1 = 6.500E+00, x2 = 2.875E+01) : \text{flux}_1 = -5.671E+04 ; \text{flux}_2 = 6.805E+04 \]

16 ( type = QS4 ) :

\[ @ (x1 = 9.500E+00, x2 = 1.250E+00) : \text{flux}_1 = 5.309E+04 ; \text{flux}_2 = -6.371E+04 \]

17 ( type = QS4 ) :

\[ @ (x1 = 9.500E+00, x2 = 3.750E+00) : \text{flux}_1 = 1.182E+05 ; \text{flux}_2 = -1.443E+04 \]

18 ( type = QS4 ) :

\[ @ (x1 = 9.500E+00, x2 = 6.429E+00) : \text{flux}_1 = 1.284E+05 ; \text{flux}_2 = 1.894E+03 \]

19 ( type = QS4 ) :

\[ @ (x1 = 9.500E+00, x2 = 9.286E+00) : \text{flux}_1 = 1.264E+05 ; \text{flux}_2 = 2.768E+02 \]

20 ( type = QS4 ) :

\[ @ (x1 = 9.500E+00, x2 = 1.214E+01) : \text{flux}_1 = 1.261E+05 ; \text{flux}_2 = 3.717E+01 \]

21 ( type = QS4 ) :

\[ @ (x1 = 9.500E+00, x2 = 1.500E+01) : \text{flux}_1 = 1.260E+05 ; \text{flux}_2 = -1.732E+01 \]

22 ( type = QS4 ) :

\[ @ (x1 = 9.500E+00, x2 = 1.786E+01) : \text{flux}_1 = 1.262E+05 ; \text{flux}_2 = -1.580E+02 \]

23 ( type = QS4 ) :

\[ @ (x1 = 9.500E+00, x2 = 2.071E+01) : \text{flux}_1 = 1.274E+05 ; \text{flux}_2 = -1.086E+03 \]

24 ( type = QS4 ) :

\[ @ (x1 = 9.500E+00, x2 = 2.357E+01) : \text{flux}_1 = 1.355E+05 ; \text{flux}_2 = -7.419E+03 \]

25 ( type = QS4 ) : [ ]
\( \theta(x_1 = 9.500000, x_2 = 2.625000) : \text{flux}_1 = 5.395000 \text{; flux}_2 = -7.136000 \)  

---  

\( \theta(x_1 = 1.250000, x_2 = 2.625000) : \text{flux}_1 = 8.146000 \text{; flux}_2 = -1.679000 \)  

---  

\( \theta(x_1 = 1.550000, x_2 = 2.625000) : \text{flux}_1 = 4.018000 \text{; flux}_2 = -1.533000 \)  

---  

\( \theta(x_1 = 1.850000, x_2 = 2.625000) : \text{flux}_1 = 6.187000 \text{; flux}_2 = -7.425000 \)  

---  

\( \theta(x_1 = 9.500000, x_2 = 2.875000) : \text{flux}_1 = -1.733000 \text{; flux}_2 = -1.569000 \)  

---  

\( \theta(x_1 = 1.250000, x_2 = 2.875000) : \text{flux}_1 = 8.146000 \text{; flux}_2 = -1.679000 \)  

---  

\( \theta(x_1 = 1.550000, x_2 = 2.875000) : \text{flux}_1 = 4.018000 \text{; flux}_2 = -1.533000 \)  

---  

\( \theta(x_1 = 1.850000, x_2 = 2.875000) : \text{flux}_1 = 6.187000 \text{; flux}_2 = -7.425000 \)  

maximum values of element variables:

\[ \begin{align*}  
\text{max} & | \text{flux}_1 | = 1.355E+05 @ x_1 = 9.500E+00, x_2 = 2.357E+01 \\
\text{max} & | \text{flux}_2 | = 1.679E+05 @ x_1 = 1.250E+01, x_2 = 2.875E+01 
\end{align*} \]

= NODAL QUANTITIES =

node : 1 ( x_1 = -7.523E-37, x_2 = -7.523E-37 ), phi = 6.111E-16  
node : 2 ( x_1 = 2.500E+00, x_2 = 7.523E-37 ), phi = 1.562E-15  
node : 3 ( x_1 = 5.000E+00, x_2 = 7.521E-37 ), phi = 3.451E-15  
node : 4 ( x_1 = 8.000E+00, x_2 = 7.521E-37 ), phi = 3.405E-15
node :  5 ( x1 =  1.100E+01,  x2 =  7.521E-37 ), phi =  1.593E-15
node :  6 ( x1 = -1.505E+36,  x2 =  2.500E+00 ), phi =  6.111E-16
node :  7 ( x1 =  2.500E+00,  x2 =  2.500E+00 ), phi =  1.222E+05
node : 10 ( x1 =  2.500E+00,  x2 =  2.500E+00 ), phi =  2.684E-15
node : 11 ( x1 = -3.762E+37,  x2 =  5.000E+00 ), phi =  6.111E-16
node : 12 ( x1 =  2.500E+00,  x2 =  5.000E+00 ), phi =  1.562E-15
node : 13 ( x1 =  5.000E+00,  x2 =  5.000E+00 ), phi =  6.643E-15
node : 14 ( x1 =  8.000E+00,  x2 =  5.000E+00 ), phi =  3.907E+05
node : 15 ( x1 = 1.100E+01,  x2 =  5.000E+00 ), phi =  2.684E-15
node : 18 ( x1 =  5.000E+00,  x2 =  7.857E+00 ), phi =  5.466E-15
node : 19 ( x1 =  8.000E+00,  x2 =  7.857E+00 ), phi =  3.799E+05
node : 20 ( x1 =  1.100E+01,  x2 =  7.857E+00 ), phi =  5.466E-15
node : 23 ( x1 =  5.000E+00,  x2 =  1.071E+01 ), phi =  5.404E-15
node : 24 ( x1 =  8.000E+00,  x2 =  1.071E+01 ), phi =  3.783E+05
node : 25 ( x1 =  1.100E+01,  x2 =  1.071E+01 ), phi =  5.404E-15
node : 28 ( x1 =  5.000E+00,  x2 =  1.357E+01 ), phi =  5.400E-15
node : 29 ( x1 =  8.000E+00,  x2 =  1.357E+01 ), phi =  3.782E+05
node : 30 ( x1 =  1.100E+01,  x2 =  1.357E+01 ), phi =  5.396E-15
node : 33 ( x1 =  5.000E+00,  x2 =  1.643E+01 ), phi =  5.401E-15
node : 34 ( x1 =  8.000E+00,  x2 =  1.643E+01 ), phi =  3.782E+05
node : 35 ( x1 =  1.100E+01,  x2 =  1.643E+01 ), phi =  5.400E-15
node : 38 ( x1 =  5.000E+00,  x2 =  1.929E+01 ), phi =  5.435E-15
node : 39 ( x1 =  8.000E+00,  x2 =  1.929E+01 ), phi =  3.791E+05
node : 40 ( x1 =  1.100E+01,  x2 =  1.929E+01 ), phi =  5.435E-15
node : 43 ( x1 =  5.000E+00,  x2 =  2.214E+01 ), phi =  5.678E-15
node : 44 ( x1 =  8.000E+00,  x2 =  2.214E+01 ), phi =  3.853E+05
node : 45 ( x1 =  1.100E+01,  x2 =  2.214E+01 ), phi =  5.678E-15
node : 48 ( x1 =  5.000E+00,  x2 =  2.500E+01 ), phi =  5.065E-15
node : 49 ( x1 =  8.000E+00,  x2 =  2.500E+01 ), phi =  4.277E+05
node : 50 ( x1 =  1.100E+01,  x2 =  2.500E+01 ), phi =  1.055E-14
node : 51 ( x1 =  1.400E+01,  x2 =  2.500E+01 ), phi =  7.124E-15
node : 52 ( x1 =  1.700E+01,  x2 =  2.500E+01 ), phi =  4.837E-15
node : 53 ( x1 =  2.000E+01,  x2 =  2.500E+01 ), phi =  1.856E-15
node : 54 ( x1 =  5.000E+00,  x2 =  2.750E+01 ), phi =  2.919E-15
node : 55 ( x1 =  8.000E+00,  x2 =  2.750E+01 ), phi =  3.402E+05
node : 56 ( x1 =  1.100E+01,  x2 =  2.750E+01 ), phi =  4.442E+05
node : 57 ( x1 =  1.400E+01,  x2 =  2.750E+01 ), phi =  3.954E+05
node : 58 ( x1 =  1.700E+01,  x2 =  2.750E+01 ), phi =  3.712E+05
node : 59 ( x1 =  2.000E+01,  x2 =  2.750E+01 ), phi =  8.520E-16
node : 60 ( x1 =  5.000E+00,  x2 =  3.000E+01 ), phi =  1.701B-15
node : 61 ( x1 =  8.000E+00,  x2 =  3.000E+01 ), phi =  4.843B-15
node : 62 ( x1 =  1.100E+01,  x2 =  3.000E+01 ), phi =  7.101B-15
node : 63 ( x1 =  1.400E+01,  x2 =  3.000E+01 ), phi =  7.124B-15
node : 64 ( x1 =  1.700E+01,  x2 =  3.000E+01 ), phi =  4.837B-15
node : 65 ( x1 =  2.000E+01,  x2 =  3.000E+01 ), phi =  1.856B-15

max | phi | = 4.442E+05 @ node     56 ( 1.100E+01, 2.750E+01)

ud_scalar -> end of analysis . . . . . . . .
The contour of Prandtl stress function (the primary dependent variable) is shown in Figure 3.
The mesh is next refined to one containing 128 four-node (QS4) elements. This is shown in Figure 4.

![128-Element Mesh of 4-Node (QS4) Quadrilaterals](image)

**Figure 4.** 128-Element Mesh of 4-Node (QS4) Quadrilaterals

The *UD_scalar* data file associated with this mesh is shown below. Note the similarity (in terms of number of lines used) to the file associated with the first (coarse) mesh.
anal tit "torsion of an 'S' cross-section"
anal tit "comprised of two materials"
anal tit "via Prandtl's approach"
anal tit "using 4-node quadrilateral elements"
anal tit "second mesh: 128 elements"

! analysis action analyze
analysis temporal static
!
! echo warn off
! echo nodes off
! echo elem off
!
dim max scalar_1 2
dim max nodes 221
dim max qs4 128
!
finish settings
!
scalar source number 1 source_1 4.200E+04
!
scalar source number 2 source_1 1.260E+05
!
scalar conduc constant number 1 &
   desc "this is material #1" k11 1.0 k22 1.0
!
scalar conduc constant number 2 &
   desc "this is material #2" k11 1.0 k22 1.0
!

! nodes line number 1
nodes line number 5 x1 5.0 inc 1
nodes line number 41 x1 5.0 x2 5.0 inc 9
nodes line number 37 x2 5.0 inc -1
nodes line number -1 inc -9
!

! nodes line number -5
nodes line number 9 x1 11.0 inc 1
nodes line number 45 x1 11.0 x2 5.0 inc 9
nodes line number -41 inc -1
nodes line number -5 inc -9
!

! nodes line number -41
nodes line number -45 inc 1
nodes line number 171 x1 11.0 x2 25.0 inc 9
nodes line number 167 x1 5.0 x2 25.0 inc -1
nodes line number -41 inc -9
!

! nodes line number -167
nodes line number 177 x1 20.0 x2 25.0 inc 1
nodes line number 221 x1 20.0 x2 30.0 inc 11
nodes line number 211 x1 5.0 x2 30.0 inc -1
nodes line number -167 inc -11
!
element scalar typ "qs4" nodes 1 2 11 10 scalar 1 &
   1_add 3 1_incr 1 2_add 3 2_incr 9
element scalar typ "qs4" nodes 5 6 15 14 scalar 1 &
   1_add 1 1_incr 1 2_add 17 2_incr 9
element scalar typ "qs4" nodes 167 168 179 178 scalar 1 &
1_add 1 1_incr 1 2_add 3 2_incr 11
!
element scalar typ "qs4" nodes 7 8 17 16 scalar 2 &
1_add 1 1_incr 1 2_add 17 2_incr 9
element scalar typ "qs4" nodes 169 170 181 180 scalar 2 &
1_add 7 1_incr 1 2_add 3 2_incr 11
!
generate surfaces
!
spec conc scalar nodes 1:9 18:171:9 phi value 0.0
spec conc scalar nodes 172:177 188:221:11 phi value 0.0
spec conc scalar nodes 10:37:9 38:40 phi value 0.0
spec conc scalar nodes 41:167:9 178:211:11 phi value 0.0
spec conc scalar nodes 212:220 phi value 0.0
!
finish data
!
solution time final 1.0 increments 1 output 1:10:1
!
finished loading

The resulting UD_scalar output file generated using the above data is shown below. Note that the “echo” print of node and element data has been turned “off” in this file.
torsion of an 'S' cross-section
comprised of two materials
via Prandtl's approach
using 4-node quadrilateral elements
second mesh: 128 elements

======================================================================
| DYNAMIC STORAGE ALLOCATION |
======================================================================

Largest NODE number which can used in the mesh = 221
Max. no. of CONSTANT scalar conductivity idealizations = 2
Max. no. of 4-node quad. "scalar" (QS4) elements = 128

======================================================================
= GENERAL ANALYSIS INFORMATION =
======================================================================

--> analysis with SCALAR primary dependent variables
shall be performed

---> TWO-DIMENSIONAL solution domain assumed
        (PLANE STRESS idealization)

---> solver type used : SKYLINE

---> storage type : SYMMETRIC

---> "Isoparametric" mesh generation scheme used

---> LINEAR analysis

======================================================================
= SCALAR MATERIAL PARAMETERS =
======================================================================

---> idealization no.: 1

                   type : constant scalar conductivity coefficients
info. : this is material #1

"Conductivities" (material parameters) :

           k_11 = 1.000E+00  k_12 = 0.000E+00  k_13 = 0.000E+00
           k_22 = 1.000E+00  k_23 = 0.000E+00
           k_33 = 1.000E+00

source term S_1 = 4.200E+04
source term S_2 = 0.000E+00
-> idealization no.: 2

 type: constant scalar conductivity coefficients
 info.: this is material #2

"Conductivities" (material parameters):

\[
\begin{align*}
k_{11} &= 1.000E+00 \\
k_{12} &= 0.000E+00 \\
k_{13} &= 0.000E+00 \\
k_{22} &= 1.000E+00 \\
k_{23} &= 0.000E+00 \\
k_{33} &= 1.000E+00
\end{align*}
\]

source term \(S_1\) = 1.260E+05
source term \(S_2\) = 0.000E+00

======================================================================
= NODE POINT SPECIFICATIONS =
======================================================================

| Node Number | (coordinates) specification:
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 : ( \(x_1 = -1.881E-36, x_2 = -6.395E-36\) )
    \(\phi\) (essential specification) = 0.000E+00

2 : ( \(x_1 = 1.250E+00, x_2 = 3.385E-36\) )
    \(\phi\) (essential specification) = 0.000E+00

3 : ( \(x_1 = 2.500E+00, x_2 = 5.752E-36\) )
    \(\phi\) (essential specification) = 0.000E+00

4 : ( \(x_1 = 3.750E+00, x_2 = -9.824E-37\) )
    \(\phi\) (essential specification) = 0.000E+00

5 : ( \(x_1 = 5.000E+00, x_2 = -1.008E-35\) )
    \(\phi\) (essential specification) = 0.000E+00
6 : ( x1 = 6.500E+00, x2 = 3.385E-36 )
phi (essential specification) = 0.000E+00

7 : ( x1 = 8.000E+00, x2 = 5.752E-36 )
phi (essential specification) = 0.000E+00

8 : ( x1 = 9.500E+00, x2 = -9.824E-37 )
phi (essential specification) = 0.000E+00

9 : ( x1 = 1.100E+01, x2 = -4.065E-36 )
phi (essential specification) = 0.000E+00

10 : ( x1 = -2.330E-36, x2 = 1.250E+00 )
phi (essential specification) = 0.000E+00

18 : ( x1 = 1.100E+01, x2 = 1.250E+00 )
phi (essential specification) = 0.000E+00

19 : ( x1 = 6.230E-36, x2 = 2.500E+00 )
phi (essential specification) = 0.000E+00

27 : ( x1 = 1.100E+01, x2 = 2.500E+00 )
phi (essential specification) = 0.000E+00

28 : ( x1 = 1.485E-35, x2 = 3.750E+00 )
phi (essential specification) = 0.000E+00

36 : ( x1 = 1.100E+01, x2 = 3.750E+00 )
phi (essential specification) = 0.000E+00

37 : ( x1 = -7.711E-36, x2 = 5.000E+00 )
phi (essential specification) = 0.000E+00

38 : ( x1 = 1.250E+00, x2 = 5.000E+00 )
phi (essential specification) = 0.000E+00

39 : ( x1 = 2.500E+00, x2 = 5.000E+00 )
\begin{align*}
\phi \ (\text{essential specification}) &= \ 0.000E+00 \\
40 : (x_1 = 3.750E+00, x_2 = 5.000E+00) &\quad \phi \ (\text{essential specification}) = \ 0.000E+00 \\
41 : (x_1 = 5.000E+00, x_2 = 5.000E+00) &\quad \phi \ (\text{essential specification}) = \ 0.000E+00 \\
45 : (x_1 = 1.100E+01, x_2 = 5.000E+00) &\quad \phi \ (\text{essential specification}) = \ 0.000E+00 \\
50 : (x_1 = 5.000E+00, x_2 = 6.429E+00) &\quad \phi \ (\text{essential specification}) = \ 0.000E+00 \\
54 : (x_1 = 1.100E+01, x_2 = 6.429E+00) &\quad \phi \ (\text{essential specification}) = \ 0.000E+00 \\
59 : (x_1 = 5.000E+00, x_2 = 7.857E+00) &\quad \phi \ (\text{essential specification}) = \ 0.000E+00 \\
63 : (x_1 = 1.100E+01, x_2 = 7.857E+00) &\quad \phi \ (\text{essential specification}) = \ 0.000E+00 \\
68 : (x_1 = 5.000E+00, x_2 = 9.286E+00) &\quad \phi \ (\text{essential specification}) = \ 0.000E+00 \\
72 : (x_1 = 1.100E+01, x_2 = 9.286E+00) &\quad \phi \ (\text{essential specification}) = \ 0.000E+00 \\
77 : (x_1 = 5.000E+00, x_2 = 1.071E+01) &\quad \phi \ (\text{essential specification}) = \ 0.000E+00 \\
81 : (x_1 = 1.100E+01, x_2 = 1.071E+01) &\quad \phi \ (\text{essential specification}) = \ 0.000E+00 \\
86 : (x_1 = 5.000E+00, x_2 = 1.214E+01) &\quad \phi \ (\text{essential specification}) = \ 0.000E+00
\end{align*}
90 : ( x1 = 1.100E+01, x2 = 1.214E+01 )
   phi (essential specification) = 0.000E+00

95 : ( x1 = 5.000E+00, x2 = 1.357E+01 )
   phi (essential specification) = 0.000E+00

99 : ( x1 = 1.100E+01, x2 = 1.357E+01 )
   phi (essential specification) = 0.000E+00

104 : ( x1 = 5.000E+00, x2 = 1.500E+01 )
   phi (essential specification) = 0.000E+00

108 : ( x1 = 1.100E+01, x2 = 1.500E+01 )
   phi (essential specification) = 0.000E+00

113 : ( x1 = 5.000E+00, x2 = 1.643E+01 )
   phi (essential specification) = 0.000E+00

117 : ( x1 = 1.100E+01, x2 = 1.643E+01 )
   phi (essential specification) = 0.000E+00

122 : ( x1 = 5.000E+00, x2 = 1.786E+01 )
   phi (essential specification) = 0.000E+00

126 : ( x1 = 1.100E+01, x2 = 1.786E+01 )
   phi (essential specification) = 0.000E+00

131 : ( x1 = 5.000E+00, x2 = 1.929E+01 )
   phi (essential specification) = 0.000E+00

135 : ( x1 = 1.100E+01, x2 = 1.929E+01 )
   phi (essential specification) = 0.000E+00

140 : ( x1 = 5.000E+00, x2 = 2.071E+01 )
   phi (essential specification) = 0.000E+00

144 : ( x1 = 1.100E+01, x2 = 2.071E+01 )
   phi (essential specification) = 0.000E+00
149 : ( x1 = 5.000E+00, x2 = 2.214E+01 )
  phi (essential specification) = 0.000E+00

153 : ( x1 = 1.100E+01, x2 = 2.214E+01 )
  phi (essential specification) = 0.000E+00

158 : ( x1 = 5.000E+00, x2 = 2.357E+01 )
  phi (essential specification) = 0.000E+00

162 : ( x1 = 1.100E+01, x2 = 2.357E+01 )
  phi (essential specification) = 0.000E+00

167 : ( x1 = 5.000E+00, x2 = 2.500E+01 )
  phi (essential specification) = 0.000E+00

171 : ( x1 = 1.100E+01, x2 = 2.500E+01 )
  phi (essential specification) = 0.000E+00

172 : ( x1 = 1.250E+01, x2 = 2.500E+01 )
  phi (essential specification) = 0.000E+00

173 : ( x1 = 1.400E+01, x2 = 2.500E+01 )
  phi (essential specification) = 0.000E+00

174 : ( x1 = 1.550E+01, x2 = 2.500E+01 )
  phi (essential specification) = 0.000E+00

175 : ( x1 = 1.700E+01, x2 = 2.500E+01 )
  phi (essential specification) = 0.000E+00

176 : ( x1 = 1.850E+01, x2 = 2.500E+01 )
  phi (essential specification) = 0.000E+00

177 : ( x1 = 2.000E+01, x2 = 2.500E+01 )
  phi (essential specification) = 0.000E+00

178 : ( x1 = 5.000E+00, x2 = 2.625E+01 )
\[
\phi \text{ (essential specification)} = 0.000E+00
\]

188 : ( x1 = 2.000E+01, x2 = 2.625E+01 )
\[
\phi \text{ (essential specification)} = 0.000E+00
\]

189 : ( x1 = 5.000E+00, x2 = 2.750E+01 )
\[
\phi \text{ (essential specification)} = 0.000E+00
\]

199 : ( x1 = 2.000E+01, x2 = 2.750E+01 )
\[
\phi \text{ (essential specification)} = 0.000E+00
\]

200 : ( x1 = 5.000E+00, x2 = 2.875E+01 )
\[
\phi \text{ (essential specification)} = 0.000E+00
\]

210 : ( x1 = 2.000E+01, x2 = 2.875E+01 )
\[
\phi \text{ (essential specification)} = 0.000E+00
\]

211 : ( x1 = 5.000E+00, x2 = 3.000E+01 )
\[
\phi \text{ (essential specification)} = 0.000E+00
\]

212 : ( x1 = 6.500E+00, x2 = 3.000E+01 )
\[
\phi \text{ (essential specification)} = 0.000E+00
\]

213 : ( x1 = 8.000E+00, x2 = 3.000E+01 )
\[
\phi \text{ (essential specification)} = 0.000E+00
\]

214 : ( x1 = 9.500E+00, x2 = 3.000E+01 )
\[
\phi \text{ (essential specification)} = 0.000E+00
\]

215 : ( x1 = 1.100E+01, x2 = 3.000E+01 )
\[
\phi \text{ (essential specification)} = 0.000E+00
\]

216 : ( x1 = 1.250E+01, x2 = 3.000E+01 )
\[
\phi \text{ (essential specification)} = 0.000E+00
\]

217 : ( x1 = 1.400E+01, x2 = 3.000E+01 )
\[
\phi \text{ (essential specification)} = 0.000E+00
\]
218 : ( x1 = 1.550E+01, x2 = 3.000E+01 )
    phi (essential specification) = 0.000E+00

219 : ( x1 = 1.700E+01, x2 = 3.000E+01 )
    phi (essential specification) = 0.000E+00

220 : ( x1 = 1.850E+01, x2 = 3.000E+01 )
    phi (essential specification) = 0.000E+00

221 : ( x1 = 2.000E+01, x2 = 3.000E+01 )
    phi (essential specification) = 0.000E+00

~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
end of mathematical model data
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

======================================================================
=                  E L E M E N T   F L U X E S                     =
======================================================================

--> element 1 ( type = QS4 ) : [ ]

............... 
@ (x1 = 6.250E-01, x2 = 6.250E-01) : flux_1 = -2.405E+04 ; flux_2 = -2.405E+04

--> element 2 ( type = QS4 ) : [ ]

............... 
@ (x1 = 1.875E+00, x2 = 6.250E-01) : flux_1 = -1.156E+04 ; flux_2 = -5.966E+04

--> element 3 ( type = QS4 ) : [ ]

..................
\( (x_1 = 3.125E+00, x_2 = 6.250E-01) : \text{flux}_1 = -8.084E+03 \ ; \text{flux}_2 = -7.930E+04 \)

\[ \text{element} \quad 4 \ (\text{type} = \text{QS4}) : \text{flux}_1 = -8.084E+03 \ ; \text{flux}_2 = -7.930E+04 \]

\[ \text{element} \quad 5 \ (\text{type} = \text{QS4}) : \text{flux}_1 = -9.556E+04 \]

\[ \text{element} \quad 6 \ (\text{type} = \text{QS4}) : \text{flux}_1 = -6.661E+03 \]

\[ \text{element} \quad 7 \ (\text{type} = \text{QS4}) : \text{flux}_1 = -2.819E+04 \ ; \text{flux}_2 = -1.840E+04 \]

\[ \text{element} \quad 8 \ (\text{type} = \text{QS4}) : \text{flux}_1 = -2.352E+04 \ ; \text{flux}_2 = -4.040E+04 \]

\[ \text{element} \quad 9 \ (\text{type} = \text{QS4}) : \text{flux}_1 = -2.104E+04 \ ; \text{flux}_2 = -2.836E+04 \]

\[ \text{element} \quad 10 \ (\text{type} = \text{QS4}) : \text{flux}_1 = -5.480E+04 \ ; \text{flux}_2 = -6.629E+03 \]
@ (x1 = 1.875E+00, x2 = 3.125E+00 ) : flux_1 = -2.842E+04 ; flux_2 = 1.811E+04

--> element 11 ( type = QS4 ) : [ ]

........................................

@ (x1 = 3.125E+00, x2 = 3.125E+00 ) : flux_1 = -2.282E+04 ; flux_2 = 2.606E+04

--> element 12 ( type = QS4 ) : [ ]

........................................

@ (x1 = 4.375E+00, x2 = 3.125E+00 ) : flux_1 = -3.755E+04 ; flux_2 = 2.229E+04

--> element 13 ( type = QS4 ) : [ ]

........................................

@ (x1 = 6.250E+01, x2 = 4.375E+00 ) : flux_1 = -2.409E+04 ; flux_2 = 2.409E+04

--> element 14 ( type = QS4 ) : [ ]

........................................

@ (x1 = 1.875E+00, x2 = 4.375E+00 ) : flux_1 = -1.178E+04 ; flux_2 = 5.995E+04

--> element 15 ( type = QS4 ) : [ ]

........................................

@ (x1 = 3.125E+00, x2 = 4.375E+00 ) : flux_1 = -9.866E+03 ; flux_2 = 8.160E+04

--> element 16 ( type = QS4 ) : [ ]

........................................

@ (x1 = 4.375E+00, x2 = 4.375E+00 ) : flux_1 = -2.221E+04 ; flux_2 = 1.137E+05

--> element 17 ( type = QS4 ) : [ ]

........................................
@ (x1 = 5.750E+00, x2 = 6.250E-01) : \text{flux}_1 = -9.713E+03 \; ; \; \text{flux}_2 = -1.154E+05

--> element 18 ( type = QS4 ) : [ ]

...........................
@ (x1 = 7.250E+00, x2 = 6.250E-01) : \text{flux}_1 = -1.491E+04 \; ; \; \text{flux}_2 = -1.449E+05

--> element 19 ( type = QS4 ) : [ ]

...........................
@ (x1 = 5.750E+00, x2 = 1.875E+00) : \text{flux}_1 = -2.950E+04 \; ; \; \text{flux}_2 = -5.965E+04

--> element 20 ( type = QS4 ) : [ ]

...........................
@ (x1 = 7.250E+00, x2 = 1.875E+00) : \text{flux}_1 = -3.646E+04 \; ; \; \text{flux}_2 = -7.970E+04

--> element 21 ( type = QS4 ) : [ ]

...........................
@ (x1 = 5.750E+00, x2 = 3.125E+00) : \text{flux}_1 = -5.266E+04 \; ; \; \text{flux}_2 = -2.858E+02

--> element 22 ( type = QS4 ) : [ ]

...........................
@ (x1 = 7.250E+00, x2 = 3.125E+00) : \text{flux}_1 = -5.319E+04 \; ; \; \text{flux}_2 = -2.812E+04

--> element 23 ( type = QS4 ) : [ ]

...........................
@ (x1 = 5.750E+00, x2 = 4.375E+00) : \text{flux}_1 = -1.207E+05 \; ; \; \text{flux}_2 = 6.997E+04

--> element 24 ( type = QS4 ) : [ ]
\[
\theta(x_1 = 7.250E+00, x_2 = 4.375E+00) : \text{flux}_1 = -7.038E+04 ; \text{flux}_2 = -4.448E+03
\]

\[
\rightarrow \text{element} \quad 25 \quad \{ \text{type} = \text{QS4} \} : \quad [ ]
\]

\[
\theta(x_1 = 5.750E+00, x_2 = 5.714E+00) : \text{flux}_1 = -1.682E+05 ; \text{flux}_2 = 7.763E+03
\]

\[
\rightarrow \text{element} \quad 26 \quad \{ \text{type} = \text{QS4} \} : \quad [ ]
\]

\[
\theta(x_1 = 7.250E+00, x_2 = 5.714E+00) : \text{flux}_1 = -1.597E+05 ; \text{flux}_2 = 1.181E+03
\]

\[
\rightarrow \text{element} \quad 27 \quad \{ \text{type} = \text{QS4} \} : \quad [ ]
\]

\[
\theta(x_1 = 7.250E+00, x_2 = 7.143E+00) : \text{flux}_1 = -9.426E+04 ; \text{flux}_2 = 1.996E+03
\]

\[
\rightarrow \text{element} \quad 28 \quad \{ \text{type} = \text{QS4} \} : \quad [ ]
\]

\[
\theta(x_1 = 5.750E+00, x_2 = 8.571E+00) : \text{flux}_1 = -1.582E+05 ; \text{flux}_2 = 3.428E+02
\]

\[
\rightarrow \text{element} \quad 29 \quad \{ \text{type} = \text{QS4} \} : \quad [ ]
\]

\[
\theta(x_1 = 5.750E+00, x_2 = 8.571E+00) : \text{flux}_1 = -9.462E+04 ; \text{flux}_2 = 6.767E+02
\]

\[
\rightarrow \text{element} \quad 30 \quad \{ \text{type} = \text{QS4} \} : \quad [ ]
\]

\[
\theta(x_1 = 7.250E+00, x_2 = 8.571E+00) : \text{flux}_1 = -9.462E+04 ; \text{flux}_2 = 6.767E+02
\]

\[
\rightarrow \text{element} \quad 31 \quad \{ \text{type} = \text{QS4} \} : \quad [ ]
\]

\[
\text{------------------------------------------}
\]
@ (x1 = 5.750E+00, x2 = 1.000E+01) : flux_1 = -1.578E+05 ; flux_2 = 1.212E+02

--> element 32 ( type = QS4 ) : [ ]

.........................

@ (x1 = 7.250E+00, x2 = 1.000E+01) : flux_1 = -9.460E+04 ; flux_2 = 2.712E+02

--> element 33 ( type = QS4 ) : [ ]

.........................

@ (x1 = 5.750E+00, x2 = 1.143E+01) : flux_1 = -1.576E+05 ; flux_2 = 4.590E+01

--> element 34 ( type = QS4 ) : [ ]

.........................

@ (x1 = 7.250E+00, x2 = 1.143E+01) : flux_1 = -9.456E+04 ; flux_2 = 1.077E+02

--> element 35 ( type = QS4 ) : [ ]

.........................

@ (x1 = 5.750E+00, x2 = 1.286E+01) : flux_1 = -1.576E+05 ; flux_2 = 1.018E+01

--> element 36 ( type = QS4 ) : [ ]

.........................

@ (x1 = 7.250E+00, x2 = 1.286E+01) : flux_1 = -9.454E+04 ; flux_2 = 2.412E+01

--> element 37 ( type = QS4 ) : [ ]

.........................

@ (x1 = 5.750E+00, x2 = 1.429E+01) : flux_1 = -1.576E+05 ; flux_2 = -1.748E+01

--> element 38 ( type = QS4 ) : [ ]

............................
\[ (x_1 = 7.250\times10^0, x_2 = 1.429\times10^1 ) : \text{flux}_1 = -9.454\times10^4 ; \text{flux}_2 = -4.228\times10^1 \]

--> element 39 ( type = QS4 ) : [ ]

\[ (x_1 = 5.750\times10^0, x_2 = 1.571\times10^1 ) : \text{flux}_1 = -1.577\times10^5 ; \text{flux}_2 = -5.627\times10^1 \]

--> element 40 ( type = QS4 ) : [ ]

\[ (x_1 = 7.250\times10^0, x_2 = 1.571\times10^1 ) : \text{flux}_1 = -9.457\times10^4 ; \text{flux}_2 = -1.360\times10^2 \]

--> element 41 ( type = QS4 ) : [ ]

\[ (x_1 = 5.750\times10^0, x_2 = 1.714\times10^1 ) : \text{flux}_1 = -1.578\times10^5 ; \text{flux}_2 = -1.313\times10^2 \]

--> element 42 ( type = QS4 ) : [ ]

\[ (x_1 = 7.250\times10^0, x_2 = 1.714\times10^1 ) : \text{flux}_1 = -9.464\times10^4 ; \text{flux}_2 = -3.181\times10^2 \]

--> element 43 ( type = QS4 ) : [ ]

\[ (x_1 = 5.750\times10^0, x_2 = 1.857\times10^1 ) : \text{flux}_1 = -1.582\times10^5 ; \text{flux}_2 = -2.890\times10^2 \]

--> element 44 ( type = QS4 ) : [ ]

\[ (x_1 = 7.250\times10^0, x_2 = 1.857\times10^1 ) : \text{flux}_1 = -9.482\times10^4 ; \text{flux}_2 = -7.046\times10^2 \]

--> element 45 ( type = QS4 ) : [ ]
\@
\(x_1 = 5.750\times10^0, x_2 = 2.000\times10^1\) : \(\text{flux}_1 = -1.591\times10^5\); \(\text{flux}_2 = -6.140\times10^2\)

\(->\) element 46 ( type = QS4 ) : [ ]


\@
\(x_1 = 7.250\times10^0, x_2 = 2.000\times10^1\) : \(\text{flux}_1 = -9.523\times10^4\); \(\text{flux}_2 = -1.530\times10^3\)

\(->\) element 47 ( type = QS4 ) : [ ]


\@
\(x_1 = 5.750\times10^0, x_2 = 2.143\times10^1\) : \(\text{flux}_1 = -1.608\times10^5\); \(\text{flux}_2 = -1.198\times10^3\)

\(->\) element 48 ( type = QS4 ) : [ ]


\@
\(x_1 = 7.250\times10^0, x_2 = 2.143\times10^1\) : \(\text{flux}_1 = -9.629\times10^4\); \(\text{flux}_2 = -3.207\times10^3\)

\(->\) element 49 ( type = QS4 ) : [ ]


\@
\(x_1 = 5.750\times10^0, x_2 = 2.286\times10^1\) : \(\text{flux}_1 = -1.634\times10^5\); \(\text{flux}_2 = -1.448\times10^3\)

\(->\) element 50 ( type = QS4 ) : [ ]


\@
\(x_1 = 7.250\times10^0, x_2 = 2.286\times10^1\) : \(\text{flux}_1 = -1.001\times10^5\); \(\text{flux}_2 = -6.036\times10^2\)

\(->\) element 51 ( type = QS4 ) : [ ]


\@
\(x_1 = 5.750\times10^0, x_2 = 2.429\times10^1\) : \(\text{flux}_1 = -1.648\times10^5\); \(\text{flux}_2 = -5.957\times10^1\)

\(->\) element 52 ( type = QS4 ) : [ ]
\[ (x_1 = 7.250\times 10^0, \ x_2 = 2.429\times 10^1) : \ \text{flux}_1 = -1.070\times 10^5 ; \ \text{flux}_2 = -4.273\times 10^3 \]

\[ \rightarrow \ \text{element} \ \ 53 \ ( \ \text{type} = \text{QS4} \ ) : \ [ \]

\[ \text{---------------------------} \]

\[ (x_1 = 5.750\times 10^0, \ x_2 = 2.562\times 10^1) : \ \text{flux}_1 = -1.607\times 10^5 ; \ \text{flux}_2 = 4.957\times 10^3 \]

\[ \rightarrow \ \text{element} \ \ 54 \ ( \ \text{type} = \text{QS4} \ ) : \ [ \]

\[ \text{---------------------------} \]

\[ (x_1 = 7.250\times 10^0, \ x_2 = 2.562\times 10^1) : \ \text{flux}_1 = -1.135\times 10^5 ; \ \text{flux}_2 = 6.902\times 10^3 \]

\[ \rightarrow \ \text{element} \ \ 55 \ ( \ \text{type} = \text{QS4} \ ) : \ [ \]

\[ \text{---------------------------} \]

\[ (x_1 = 5.750\times 10^0, \ x_2 = 2.688\times 10^1) : \ \text{flux}_1 = -1.446\times 10^5 ; \ \text{flux}_2 = 1.433\times 10^4 \]

\[ \rightarrow \ \text{element} \ \ 56 \ ( \ \text{type} = \text{QS4} \ ) : \ [ \]

\[ \text{---------------------------} \]

\[ (x_1 = 7.250\times 10^0, \ x_2 = 2.688\times 10^1) : \ \text{flux}_1 = -1.108\times 10^5 ; \ \text{flux}_2 = 3.487\times 10^4 \]

\[ \rightarrow \ \text{element} \ \ 57 \ ( \ \text{type} = \text{QS4} \ ) : \ [ \]

\[ \text{---------------------------} \]

\[ (x_1 = 5.750\times 10^0, \ x_2 = 2.813\times 10^1) : \ \text{flux}_1 = -1.081\times 10^5 ; \ \text{flux}_2 = 2.947\times 10^4 \]

\[ \rightarrow \ \text{element} \ \ 58 \ ( \ \text{type} = \text{QS4} \ ) : \ [ \]

\[ \text{---------------------------} \]

\[ (x_1 = 7.250\times 10^0, \ x_2 = 2.813\times 10^1) : \ \text{flux}_1 = -8.833\times 10^4 ; \ \text{flux}_2 = 7.968\times 10^4 \]

\[ \rightarrow \ \text{element} \ \ 59 \ ( \ \text{type} = \text{QS4} \ ) : \ [ \]

\[ \text{---------------------------} \]
\[ x_1 = 5.750 \times 10^0 \text{, } x_2 = 2.938 \times 10^1 \] : \[ \text{flux}_1 = -4.179 \times 10^4 \text{ ; } \text{flux}_2 = 5.015 \times 10^4 \]

\[ x_1 = 7.250 \times 10^0 \text{, } x_2 = 2.938 \times 10^1 \] : \[ \text{flux}_1 = -3.552 \times 10^4 \text{ ; } \text{flux}_2 = 1.429 \times 10^5 \]

\[ x_1 = 8.750 \times 10^0 \text{, } x_2 = 6.250 \times 10^1 \] : \[ \text{flux}_1 = 4.377 \times 10^3 \text{ ; } \text{flux}_2 = -1.576 \times 10^5 \]

\[ x_1 = 1.025 \times 10^1 \text{, } x_2 = 6.250 \times 10^1 \] : \[ \text{flux}_1 = 6.347 \times 10^4 \text{ ; } \text{flux}_2 = -7.617 \times 10^4 \]

\[ x_1 = 8.750 \times 10^0 \text{, } x_2 = 1.875 \times 10^0 \] : \[ \text{flux}_1 = 1.798 \times 10^4 \text{ ; } \text{flux}_2 = -7.659 \times 10^4 \]

\[ x_1 = 1.025 \times 10^1 \text{, } x_2 = 1.875 \times 10^0 \] : \[ \text{flux}_1 = 1.542 \times 10^5 \text{ ; } \text{flux}_2 = -3.276 \times 10^4 \]

\[ x_1 = 8.750 \times 10^0 \text{, } x_2 = 3.125 \times 10^0 \] : \[ \text{flux}_1 = 3.149 \times 10^4 \text{ ; } \text{flux}_2 = -3.509 \times 10^4 \]

\[ x_1 = 1.025 \times 10^1 \text{, } x_2 = 3.125 \times 10^0 \] : \[ \text{flux}_1 = 3.149 \times 10^4 \text{ ; } \text{flux}_2 = -3.509 \times 10^4 \]
\[@(x_1 = 1.025E+01, x_2 = 3.125E+00) : \text{flux}_1 = 1.940E+05 ; \text{flux}_2 = -1.497E+04\]

--> element 67 ( type = QS4 ) : [ ]

.................................
\[@(x_1 = 8.750E+00, x_2 = 4.375E+00) : \text{flux}_1 = 3.593E+04 ; \text{flux}_2 = -1.278E+04\]

--> element 68 ( type = QS4 ) : [ ]

.................................
\[@(x_1 = 1.025E+01, x_2 = 4.375E+00) : \text{flux}_1 = 2.117E+05 ; \text{flux}_2 = -6.308E+03\]

--> element 69 ( type = QS4 ) : [ ]

.................................
\[@(x_1 = 8.750E+00, x_2 = 5.714E+00) : \text{flux}_1 = 3.507E+04 ; \text{flux}_2 = -2.789E+03\]

--> element 70 ( type = QS4 ) : [ ]

.................................
\[@(x_1 = 1.025E+01, x_2 = 5.714E+00) : \text{flux}_1 = 2.188E+05 ; \text{flux}_2 = -1.918E+03\]

--> element 71 ( type = QS4 ) : [ ]

.................................
\[@(x_1 = 8.750E+00, x_2 = 7.143E+00) : \text{flux}_1 = 3.307E+04 ; \text{flux}_2 = 5.787E+02\]

--> element 72 ( type = QS4 ) : [ ]

.................................
\[@(x_1 = 1.025E+01, x_2 = 7.143E+00) : \text{flux}_1 = 2.209E+05 ; \text{flux}_2 = -2.356E+02\]

--> element 73 ( type = QS4 ) : [ ]

.................................
\@(x_1 = 8.750E+00, x_2 = 8.571E+00 \) : \text{flux}_1 = 3.188E+04 ; \text{flux}_2 = 4.693E+02

\--> element 74 ( \text{type} = \text{QS4} \) : [

\]  

\]  

\@(x_1 = 1.025E+01, x_2 = 8.571E+00 \) : \text{flux}_1 = 2.210E+05 ; \text{flux}_2 = 1.354E+02

\--> element 75 ( \text{type} = \text{QS4} \) : [

\]  

\]  

\@(x_1 = 8.750E+00, x_2 = 1.000E+01 \) : \text{flux}_1 = 3.164E+04 ; \text{flux}_2 = 2.408E+02

\--> element 76 ( \text{type} = \text{QS4} \) : [

\]  

\]  

\@(x_1 = 1.025E+01, x_2 = 1.000E+01 \) : \text{flux}_1 = 2.208E+05 ; \text{flux}_2 = 9.081E+01

\--> element 77 ( \text{type} = \text{QS4} \) : [

\]  

\]  

\@(x_1 = 8.750E+00, x_2 = 1.143E+01 \) : \text{flux}_1 = 3.156E+04 ; \text{flux}_2 = 1.032E+02

\--> element 78 ( \text{type} = \text{QS4} \) : [

\]  

\]  

\@(x_1 = 1.025E+01, x_2 = 1.143E+01 \) : \text{flux}_1 = 2.206E+05 ; \text{flux}_2 = 4.146E+01

\--> element 79 ( \text{type} = \text{QS4} \) : [

\]  

\]  

\@(x_1 = 8.750E+00, x_2 = 1.286E+01 \) : \text{flux}_1 = 3.154E+04 ; \text{flux}_2 = 2.346E+01

\--> element 80 ( \text{type} = \text{QS4} \) : [

\]  

\]
<table>
<thead>
<tr>
<th>Element</th>
<th>Type</th>
<th>Coordinates</th>
<th>Flux 1</th>
<th>Flux 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>81</td>
<td>QS4</td>
<td>(x1 = 1.025E+01, x2 = 1.286E+01)</td>
<td>2.206E+05</td>
<td>9.527E+00</td>
</tr>
<tr>
<td>82</td>
<td>QS4</td>
<td>(x1 = 8.750E+00, x2 = 1.429E+01)</td>
<td>3.154E+04</td>
<td>-4.241E+01</td>
</tr>
<tr>
<td>83</td>
<td>QS4</td>
<td>(x1 = 8.750E+00, x2 = 1.571E+01)</td>
<td>3.157E+04</td>
<td>-1.362E+02</td>
</tr>
<tr>
<td>84</td>
<td>QS4</td>
<td>(x1 = 1.025E+01, x2 = 1.571E+01)</td>
<td>2.206E+05</td>
<td>-5.650E+01</td>
</tr>
<tr>
<td>85</td>
<td>QS4</td>
<td>(x1 = 8.750E+00, x2 = 1.714E+01)</td>
<td>3.164E+04</td>
<td>-3.195E+02</td>
</tr>
<tr>
<td>86</td>
<td>QS4</td>
<td>(x1 = 1.025E+01, x2 = 1.714E+01)</td>
<td>2.207E+05</td>
<td>-1.328E+02</td>
</tr>
<tr>
<td>87</td>
<td>QS4</td>
<td>(x1 = 8.750E+00, x2 = 1.714E+01)</td>
<td>3.164E+04</td>
<td>-3.195E+02</td>
</tr>
</tbody>
</table>
\[
\theta(x_1 = 8.750E+00, x_2 = 1.857E+01) : \text{flux}_1 = 3.181E+04 ; \text{flux}_2 = -7.145E+02
\]

\[
\text{--- element 88 ( type = QS4 ) : [}
\]

\[
\theta(x_1 = 1.025E+01, x_2 = 1.857E+01) : \text{flux}_1 = 2.213E+05 ; \text{flux}_2 = -2.988E+02
\]

\[
\text{--- element 89 ( type = QS4 ) : [}
\]

\[
\theta(x_1 = 8.750E+00, x_2 = 2.000E+01) : \text{flux}_1 = 3.214E+04 ; \text{flux}_2 = -1.597E+03
\]

\[
\text{--- element 90 ( type = QS4 ) : [}
\]

\[
\theta(x_1 = 1.025E+01, x_2 = 2.000E+01) : \text{flux}_1 = 2.222E+05 ; \text{flux}_2 = -6.812E+02
\]

\[
\text{--- element 91 ( type = QS4 ) : [}
\]

\[
\theta(x_1 = 8.750E+00, x_2 = 2.143E+01) : \text{flux}_1 = 3.270E+04 ; \text{flux}_2 = -3.667E+03
\]

\[
\text{--- element 92 ( type = QS4 ) : [}
\]

\[
\theta(x_1 = 1.025E+01, x_2 = 2.143E+01) : \text{flux}_1 = 2.244E+05 ; \text{flux}_2 = -1.657E+03
\]

\[
\text{--- element 93 ( type = QS4 ) : [}
\]

\[
\theta(x_1 = 8.750E+00, x_2 = 2.286E+01) : \text{flux}_1 = 3.304E+04 ; \text{flux}_2 = -9.175E+03
\]

\[
\text{--- element 94 ( type = QS4 ) : [}
\]
@ (x1 = 1.025E+01, x2 = 2.286E+01 ) : flux_1 = 2.304E+05 ; flux_2 = -4.587E+03

--> element 95 ( type = QS4 ) : [ ]

...............................
@ (x1 = 8.750E+00, x2 = 2.429E+01 ) : flux_1 = 1.657E+04 ; flux_2 = -2.572E+04

--> element 96 ( type = QS4 ) : [ ]

...............................
@ (x1 = 1.025E+01, x2 = 2.429E+01 ) : flux_1 = 2.552E+05 ; flux_2 = -2.151E+04

--> element 97 ( type = QS4 ) : [ ]

...............................
@ (x1 = 8.750E+00, x2 = 2.562E+01 ) : flux_1 = -1.322E+04 ; flux_2 = -1.209E+04

--> element 98 ( type = QS4 ) : [ ]

...............................
@ (x1 = 1.025E+01, x2 = 2.562E+01 ) : flux_1 = 1.626E+05 ; flux_2 = -1.637E+05

--> element 99 ( type = QS4 ) : [ ]

...............................
@ (x1 = 1.175E+01, x2 = 2.562E+01 ) : flux_1 = 2.158E+04 ; flux_2 = -2.735E+05

--> element 100 ( type = QS4 ) : [ ]

...............................
@ (x1 = 1.325E+01, x2 = 2.562E+01 ) : flux_1 = 4.537E+03 ; flux_2 = -2.422E+05

--> element 101 ( type = QS4 ) : [ ]

...............................

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\(\theta(x_1 = 1.475E+01, x_2 = 2.562E+01) : \) flux_1 = 4.167E+03 ; flux_2 = -2.317E+05

\(--\rightarrow\) element 102 ( type = QS4 ) : [ ]

\(--\rightarrow\) element 103 ( type = QS4 ) : [ ]

\(--\rightarrow\) element 104 ( type = QS4 ) : [ ]

\(--\rightarrow\) element 105 ( type = QS4 ) : [ ]

\(--\rightarrow\) element 106 ( type = QS4 ) : [ ]

\(--\rightarrow\) element 107 ( type = QS4 ) : [ ]

\(--\rightarrow\) element 108 ( type = QS4 ) : [ ]
\( @ (x_1 = 1.325E+01, x_2 = 2.687E+01) : \text{flux}_1 = 1.097E+04 ; \text{flux}_2 = -8.096E+04 \)

---

\( \rightarrow \) element 109 ( type = QS4 ) :

\( \)  

------------------------

\( @ (x_1 = 1.475E+01, x_2 = 2.687E+01) : \text{flux}_1 = 9.949E+03 ; \text{flux}_2 = -7.674E+04 \)

---

\( \rightarrow \) element 110 ( type = QS4 ) :

\( \)  

------------------------

\( @ (x_1 = 1.625E+01, x_2 = 2.687E+01) : \text{flux}_1 = 2.052E+04 ; \text{flux}_2 = -7.065E+04 \)

---

\( \rightarrow \) element 111 ( type = QS4 ) :

\( \)  

------------------------

\( @ (x_1 = 1.775E+01, x_2 = 2.687E+01) : \text{flux}_1 = 5.399E+04 ; \text{flux}_2 = -5.456E+04 \)

---

\( \rightarrow \) element 112 ( type = QS4 ) :

\( \)  

------------------------

\( @ (x_1 = 1.925E+01, x_2 = 2.687E+01) : \text{flux}_1 = 1.456E+05 ; \text{flux}_2 = -2.131E+04 \)

---

\( \rightarrow \) element 113 ( type = QS4 ) :

\( \)  

------------------------

\( @ (x_1 = 8.750E+00, x_2 = 2.813E+01) : \text{flux}_1 = -5.272E+04 ; \text{flux}_2 = 1.063E+05 \)

---

\( \rightarrow \) element 114 ( type = QS4 ) :

\( \)  

------------------------

\( @ (x_1 = 1.025E+01, x_2 = 2.813E+01) : \text{flux}_1 = -2.124E+03 ; \text{flux}_2 = 1.065E+05 \)

---

\( \rightarrow \) element 115 ( type = QS4 ) :

\( \)  

------------------------
\[@(x_1 = 1.175E+01, x_2 = 2.813E+01) : \text{flux}_1 = 1.144E+04 ; \text{flux}_2 = 9.335E+04\]

--> element 116 ( type = QS4 ) : [
]  

.................................
\[@(x_1 = 1.325E+01, x_2 = 2.812E+01) : \text{flux}_1 = 9.886E+03 ; \text{flux}_2 = 8.242E+04\]

--> element 117 ( type = QS4 ) : [
]  

.................................
\[@(x_1 = 1.475E+01, x_2 = 2.812E+01) : \text{flux}_1 = 9.886E+03 ; \text{flux}_2 = 7.682E+04\]

--> element 118 ( type = QS4 ) : [
]  

.................................
\[@(x_1 = 1.625E+01, x_2 = 2.812E+01) : \text{flux}_1 = 2.052E+04 ; \text{flux}_2 = 7.065E+04\]

--> element 119 ( type = QS4 ) : [
]  

.................................
\[@(x_1 = 1.775E+01, x_2 = 2.812E+01) : \text{flux}_1 = 5.399E+04 ; \text{flux}_2 = 5.456E+04\]

--> element 120 ( type = QS4 ) : [
]  

.................................
\[@(x_1 = 1.925E+01, x_2 = 2.812E+01) : \text{flux}_1 = 1.456E+05 ; \text{flux}_2 = 2.131E+04\]

--> element 121 ( type = QS4 ) : [
]  

.................................
\[@(x_1 = 8.750E+00, x_2 = 2.938E+01) : \text{flux}_1 = -2.390E+04 ; \text{flux}_2 = 2.142E+05\]

--> element 122 ( type = QS4 ) : [ ]
\( @(x_1 = 1.025E+01, x_2 = 2.938E+01) : \text{flux}_1 = -3.465E+03 ; \text{flux}_2 = 2.471E+05 \)

\( \longrightarrow \) element 123 ( type = QS4 ) : [ ]

\( \longrightarrow \) element 124 ( type = QS4 ) : [ ]

\( \longrightarrow \) element 125 ( type = QS4 ) : [ ]

\( \longrightarrow \) element 126 ( type = QS4 ) : [ ]

\( \longrightarrow \) element 127 ( type = QS4 ) : [ ]

\( \longrightarrow \) element 128 ( type = QS4 ) : [ ]

maximum values of element variables:

\( \longrightarrow \) element 123 ( type = QS4 ) : [ ]

\( \longrightarrow \) element 124 ( type = QS4 ) : [ ]

\( \longrightarrow \) element 125 ( type = QS4 ) : [ ]

\( \longrightarrow \) element 126 ( type = QS4 ) : [ ]

\( \longrightarrow \) element 127 ( type = QS4 ) : [ ]

\( \longrightarrow \) element 128 ( type = QS4 ) : [ ]

\( \longrightarrow \) element 129 ( type = QS4 ) : [ ]

\( \longrightarrow \) element 130 ( type = QS4 ) : [ ]
max | flux_1 | = 2.552E+05 @ x1 = 1.025E+01, x2 = 2.429E+01
max | flux_2 | = 2.735E+05 @ x1 = 1.175E+01, x2 = 2.562E+01

==========================================

= NODAL QUANTITIES =

node :  1 ( x1 = -1.881E-36, x2 = -6.395E-36 ), phi = 3.007E-16
node :  2 ( x1 = 1.250E+00, x2 = 3.385E-36 ), phi = 7.458E-16
node :  3 ( x1 = 2.500E+00, x2 = 5.752E-36 ), phi = 1.292E-15
node :  4 ( x1 = 3.750E+00, x2 = -9.824E-37 ), phi = 1.640E-15
node :  5 ( x1 = 5.000E+00, x2 = -1.008E-35 ), phi = 2.185E-15
node :  6 ( x1 = 6.500E+00, x2 = 3.385E-36 ), phi = 2.890E-15
node :  7 ( x1 = 8.000E+00, x2 = 5.752E-36 ), phi = 3.314E-15
node :  8 ( x1 = 9.500E+00, x2 = -9.824E-37 ), phi = 2.485E-15
node :  9 ( x1 = 1.100E+01, x2 = -4.065E-36 ), phi = 9.521E-16
node : 10 ( x1 = -2.330E-36, x2 = 1.250E+00 ), phi = 6.846E-16
node : 11 ( x1 = 1.250E+00, x2 = 1.250E+00 ), phi = 6.013E+04
node : 12 ( x1 = 2.500E+00, x2 = 1.250E+00 ), phi = 8.902E+04
node : 13 ( x1 = 3.750E+00, x2 = 1.250E+00 ), phi = 1.092E+05
node : 14 ( x1 = 5.000E+00, x2 = 1.250E+00 ), phi = 1.297E+05
node : 15 ( x1 = 6.500E+00, x2 = 1.250E+00 ), phi = 1.588E+05
node : 16 ( x1 = 8.000E+00, x2 = 1.250E+00 ), phi = 2.035E+05
node : 17 ( x1 = 9.500E+00, x2 = 1.250E+00 ), phi = 1.904E+05
node : 18 ( x1 = 1.100E+01, x2 = 1.250E+00 ), phi = 1.799E-15
node : 19 ( x1 = 6.230E-36, x2 = 2.500E+00 ), phi = 9.857E-16
node : 20 ( x1 = 1.250E+00, x2 = 2.500E+00 ), phi = 7.679E+04
node : 21 ( x1 = 2.500E+00, x2 = 2.500E+00 ), phi = 1.184E+05
node : 22 ( x1 = 3.750E+00, x2 = 2.500E+00 ), phi = 1.508E+05
node : 23 ( x1 = 5.000E+00, x2 = 2.500E+00 ), phi = 1.891E+05
node : 24 ( x1 = 6.500E+00, x2 = 2.500E+00 ), phi = 2.485E+05
node : 25 ( x1 = 8.000E+00, x2 = 2.500E+00 ), phi = 3.131E+05
node : 26 ( x1 = 9.500E+00, x2 = 2.500E+00 ), phi = 2.723E+05
node : 27 ( x1 = 1.100E+01, x2 = 2.500E+00 ), phi = 3.126E-15
node : 28 ( x1 = 1.485E-35, x2 = 3.750E+00 ), phi = 6.850E-16
node : 29 ( x1 = 1.250E+00, x2 = 3.750E+00 ), phi = 6.021E+04
node : 30 ( x1 = 2.500E+00, x2 = 3.750E+00 ), phi = 8.967E+04
node : 31 ( x1 = 3.750E+00, x2 = 3.750E+00 ), phi = 1.143E+05
node : 32 ( x1 = 5.000E+00, x2 = 3.750E+00 ), phi = 1.698E+05
node : 33 ( x1 =  6.500E+00, x2 =  3.750E+00 ), phi =  2.685E+05
node : 34 ( x1 =  8.000E+00, x2 =  3.750E+00 ), phi =  3.634E+05
node : 35 ( x1 =  9.500E+00, x2 =  3.750E+00 ), phi =  3.097E+05
node : 36 ( x1 =  1.100E+01, x2 =  3.750E+00 ), phi =  3.700E+15
node : 37 ( x1 =  7.111E-36, x2 =  5.000E+00 ), phi =  3.011E-16
node : 38 ( x1 =  1.250E+00, x2 =  5.000E+00 ), phi =  7.494E-16
node : 39 ( x1 =  2.500E+00, x2 =  5.000E+00 ), phi =  1.321E-15
node : 40 ( x1 =  3.750E+00, x2 =  5.000E+00 ), phi =  1.869E-15
node : 41 ( x1 =  5.000E+00, x2 =  5.000E+00 ), phi =  5.104E-15
node : 42 ( x1 =  6.500E+00, x2 =  5.000E+00 ), phi =  2.634E+05
node : 43 ( x1 =  8.000E+00, x2 =  5.000E+00 ), phi =  3.796E+05
node : 44 ( x1 =  9.500E+00, x2 =  5.000E+00 ), phi =  3.255E+05
node : 45 ( x1 =  1.100E+01, x2 =  5.000E+00 ), phi =  4.236E-15
node : 46 ( x1 =  5.000E+00, x2 =  6.429E+00 ), phi =  3.536E-15
node : 47 ( x1 =  6.500E+00, x2 =  6.429E+00 ), phi =  2.412E+05
node : 48 ( x1 =  8.000E+00, x2 =  6.429E+00 ), phi =  3.310E+05
node : 49 ( x1 =  9.500E+00, x2 =  6.429E+00 ), phi =  4.699E-15
node : 50 ( x1 =  5.000E+00, x2 =  7.857E+00 ), phi =  3.406E-15
node : 51 ( x1 =  6.500E+00, x2 =  7.857E+00 ), phi =  2.379E+05
node : 52 ( x1 =  8.000E+00, x2 =  7.857E+00 ), phi =  3.798E+05
node : 53 ( x1 =  9.500E+00, x2 =  7.857E+00 ), phi =  3.317E+05
node : 54 ( x1 =  1.100E+01, x2 =  7.857E+00 ), phi =  4.727E-15
node : 55 ( x1 =  5.000E+00, x2 =  9.286E+00 ), phi =  3.383E-15
node : 56 ( x1 =  6.500E+00, x2 =  9.286E+00 ), phi =  2.369E+05
node : 57 ( x1 =  8.000E+00, x2 =  9.286E+00 ), phi =  3.784E+05
node : 58 ( x1 =  9.500E+00, x2 =  9.286E+00 ), phi =  3.309E+05
node : 59 ( x1 =  1.100E+01, x2 =  9.286E+00 ), phi =  4.722E-15
node : 60 ( x1 =  5.000E+00, x2 =  1.071E+01 ), phi =  3.376E-15
node : 61 ( x1 =  6.500E+00, x2 =  1.071E+01 ), phi =  2.365E+05
node : 62 ( x1 =  8.000E+00, x2 =  1.071E+01 ), phi =  3.782E+05
node : 63 ( x1 =  9.500E+00, x2 =  1.071E+01 ), phi =  3.309E+05
node : 64 ( x1 =  1.100E+01, x2 =  1.071E+01 ), phi =  4.722E-15
node : 65 ( x1 =  5.000E+00, x2 =  1.214E+01 ), phi =  3.374E-15
node : 66 ( x1 =  6.500E+00, x2 =  1.214E+01 ), phi =  2.364E+05
node : 67 ( x1 =  8.000E+00, x2 =  1.214E+01 ), phi =  3.782E+05
node : 68 ( x1 =  9.500E+00, x2 =  1.214E+01 ), phi =  3.309E+05
node : 69 ( x1 =  1.100E+01, x2 =  1.214E+01 ), phi =  4.722E-15
node : 70 ( x1 =  5.000E+00, x2 =  1.357E+01 ), phi =  3.373E-15
node : 71 ( x1 =  6.500E+00, x2 =  1.357E+01 ), phi =  2.364E+05
node : 72 ( x1 =  8.000E+00, x2 =  1.357E+01 ), phi =  3.782E+05
node : 73 ( x1 =  9.500E+00, x2 =  1.357E+01 ), phi =  3.309E+05
node : 74 ( x1 =  1.100E+01, x2 =  1.357E+01 ), phi =  3.011E-16
node : 75 ( x1 =  5.000E+00, x2 =  1.490E+01 ), phi =  3.372E-15
node : 76 ( x1 =  6.500E+00, x2 =  1.490E+01 ), phi =  2.364E+05
node : 77 ( x1 =  8.000E+00, x2 =  1.490E+01 ), phi =  3.782E+05
node : 78 ( x1 =  9.500E+00, x2 =  1.490E+01 ), phi =  3.309E+05
node : 79 ( x1 =  1.100E+01, x2 =  1.490E+01 ), phi =  4.722E-15
node:  99 (x1 = 1.100E+01, x2 = 1.357E+01), phi = 4.721E-15
node:  104 (x1 = 5.000E+00, x2 = 1.500E+01), phi = 3.374E-15
node:  105 (x1 = 6.500E+00, x2 = 1.500E+01), phi = 2.364E+05
node:  106 (x1 = 8.000E+00, x2 = 1.500E+01), phi = 3.782E+05
node:  107 (x1 = 9.500E+00, x2 = 1.500E+01), phi = 3.309E+05
node:  108 (x1 = 1.100E+01, x2 = 1.500E+01), phi = 4.722E-15
node:  113 (x1 = 5.000E+00, x2 = 1.643E+01), phi = 3.377E-15
node:  114 (x1 = 6.500E+00, x2 = 1.643E+01), phi = 2.366E+05
node:  115 (x1 = 8.000E+00, x2 = 1.643E+01), phi = 3.785E+05
node:  116 (x1 = 9.500E+00, x2 = 1.643E+01), phi = 3.311E+05
node:  117 (x1 = 1.100E+01, x2 = 1.643E+01), phi = 4.725E-15
node:  122 (x1 = 5.000E+00, x2 = 1.786E+01), phi = 3.383E-15
node:  123 (x1 = 6.500E+00, x2 = 1.786E+01), phi = 2.370E+05
node:  124 (x1 = 8.000E+00, x2 = 1.786E+01), phi = 3.790E+05
node:  125 (x1 = 9.500E+00, x2 = 1.786E+01), phi = 3.315E+05
node:  126 (x1 = 1.100E+01, x2 = 1.786E+01), phi = 4.732E-15
node:  131 (x1 = 5.000E+00, x2 = 1.929E+01), phi = 3.397E-15
node:  132 (x1 = 6.500E+00, x2 = 1.929E+01), phi = 2.378E+05
node:  133 (x1 = 8.000E+00, x2 = 1.929E+01), phi = 3.323E+05
node:  134 (x1 = 9.500E+00, x2 = 1.929E+01), phi = 3.343E+05
node:  135 (x1 = 1.100E+01, x2 = 1.929E+01), phi = 4.747E-15
node:  140 (x1 = 5.000E+00, x2 = 2.071E+01), phi = 3.426E-15
node:  141 (x1 = 6.500E+00, x2 = 2.071E+01), phi = 2.395E+05
node:  142 (x1 = 8.000E+00, x2 = 2.071E+01), phi = 3.828E+05
node:  143 (x1 = 9.500E+00, x2 = 2.071E+01), phi = 3.343E+05
node:  144 (x1 = 1.100E+01, x2 = 2.071E+01), phi = 4.783E-15
node:  149 (x1 = 5.000E+00, x2 = 2.214E+01), phi = 3.470E-15
node:  150 (x1 = 6.500E+00, x2 = 2.214E+01), phi = 2.430E+05
node:  151 (x1 = 8.000E+00, x2 = 2.214E+01), phi = 3.885E+05
node:  152 (x1 = 9.500E+00, x2 = 2.214E+01), phi = 3.390E+05
node:  153 (x1 = 1.100E+01, x2 = 2.214E+01), phi = 4.897E-15
node:  154 (x1 = 5.000E+00, x2 = 2.357E+01), phi = 3.506E-15
node:  155 (x1 = 6.500E+00, x2 = 2.357E+01), phi = 2.471E+05
node:  156 (x1 = 8.000E+00, x2 = 2.357E+01), phi = 4.017E+05
node:  157 (x1 = 9.500E+00, x2 = 2.357E+01), phi = 3.521E+05
node:  158 (x1 = 1.100E+01, x2 = 2.357E+01), phi = 5.266E-15
node:  159 (x1 = 5.000E+00, x2 = 2.500E+01), phi = 3.244E-15
node:  160 (x1 = 6.500E+00, x2 = 2.500E+01), phi = 2.473E+05
node:  161 (x1 = 8.000E+00, x2 = 2.500E+01), phi = 4.137E+05
node:  162 (x1 = 9.500E+00, x2 = 2.500E+01), phi = 4.135E+05
node:  163 (x1 = 1.100E+01, x2 = 2.500E+01), phi = 9.903E-15
node:  164 (x1 = 1.250E+01, x2 = 2.500E+01), phi = 5.736E-15
node : 215 ( x1 = 1.100E+01, x2 = 3.000E+01 ), phi = 5.468E-15
node : 216 ( x1 = 1.250E+01, x2 = 3.000E+01 ), phi = 5.407E-15
node : 217 ( x1 = 1.400E+01, x2 = 3.000E+01 ), phi = 5.226E-15
node : 218 ( x1 = 1.550E+01, x2 = 3.000E+01 ), phi = 4.951E-15
node : 219 ( x1 = 1.700E+01, x2 = 3.000E+01 ), phi = 4.362E-15
node : 220 ( x1 = 1.850E+01, x2 = 3.000E+01 ), phi = 2.767E-15
node : 221 ( x1 = 2.000E+01, x2 = 3.000E+01 ), phi = 9.588E-16

max | phi | = 4.486E+05 @ node 181 ( 9.500E+00, 2.625E+01)

ud_scalar -> end of analysis . . . . . . . . .
The contour of Prandtl stress function (the primary dependent variable) is shown in Figure 5.

**Figure 5.** Contour Plot of Prandtl Stress Function Associated with 128-Element Mesh