CIEG 212
Solution for Homework Assignment #9
5.82 A steel pipe of 4-in. diameter is to support the loading shown. Knowing that the stock of pipes available has thicknesses varying from $\frac{1}{4}$ in. to 1 in. in $\frac{1}{8}$-in. increments, and that the allowable normal stress for the steel used is 24 ksi, determine the minimum wall thickness $t$ that can be used.

Shear:
$A$ to $B$ \hspace{10pt} $V = -500 \text{ lb} = -0.5 \text{ kip}$
$B$ to $C$ \hspace{10pt} $V = -500 - 500 = -1000 \text{ lb} = -1.0 \text{ kip}$

Areas:
$A$ to $B$ \hspace{10pt} $(4)(-0.5) = -2.0 \text{ kip-ft}$
$B$ to $C$ \hspace{10pt} $(4)(-1.0) = -4.0 \text{ kip-ft}$

Bending moments:
$M_A = 0$
$M_B = 0 - 2.0 = -2.0 \text{ kip-ft}$
$M_C = -2.0 - 4.0 = -6.0 \text{ kip-ft}$

Maximum $|M| = 6.0 \text{ kip-ft} = 72 \text{ kip-in.}$

$G_{\text{all}} = 24 \text{ ksi}$

$S_{\text{min}} = \frac{|M|}{G_{\text{all}}} = \frac{72}{24} = 3 \text{ in}^3$

$I = \frac{\pi}{4} (C_z^4 - C_1^4)$ \hspace{10pt} $c = C_z$ \hspace{10pt} $C_z = \frac{1}{2} d = 2.0 \text{ in.}$

$S = \frac{T}{E} = \frac{\pi}{4} \frac{C_z^4 - C_1^4}{C_z} = \frac{\pi}{4} \frac{2^4 - C_1^4}{2} = 3 \text{ in}^3$

$C_1^4 = 2^4 - \frac{(4)(2)(3)}{3} = 2.3606 \text{ in}^4 \hspace{10pt} C_1 = 1.7004 \text{ in.}$

$t_{\text{min}} = C_z - C_1 = 2.0 - 1.7004 = 0.2996 \text{ in.}$

Using $\frac{1}{8}$ in. increments for design $t = \frac{3}{8} \text{ in.}$
Problem 5.85

5.85 Determine the allowable value of P for the loading shown, knowing that the allowable normal stress is +8 ksi in tension and -18 ksi in compression.

Reactions. $B = D = 1.5 P$

Shear diagram. $A$ to $B$ $V = -P$
$B$ to $C$ $V = -P + 1.5 P = 0.5 P$
$C$ to $D$ $V = 0.5 P - P = -0.5 P$
$D$ to $E$ $V = -0.5 P + 1.5 P = P$

Areas. $A$ to $B$ $(10)(-P) = -10 P$
$B$ to $C$ $(60)(0.5 P) = 30 P$
$C$ to $D$ $(20)(-0.5 P) = -30 P$
$D$ to $E$ $(10)(P) = 10 P$

Bending moments. $M_A = 0$
$M_B = 0 - 10 P = -10 P$
$M_C = -10 P + 30 P = 20 P$
$M_D = 20 P - 30 P = -10 P$
$M_E = -10 P + 10 P = 0$

Largest positive bending moment = 20 P
Largest negative bending moment = -10 P

Centroid and moment of inertia.

<table>
<thead>
<tr>
<th>Part</th>
<th>$A_o$ in$^2$</th>
<th>$x_o$ in</th>
<th>$A_d x_o$ in$^3$</th>
<th>$d$ in</th>
<th>$A_d$ in$^2$</th>
<th>$I$ in$^4$</th>
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<td>1.75</td>
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<tr>
<td>Z</td>
<td>12</td>
<td>21</td>
<td></td>
<td></td>
<td></td>
<td>26.25</td>
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</tbody>
</table>

$ar{y} = \frac{21}{12} = 1.75$ in.
$I = \Sigma A d^2 + \Sigma I = 37.25$ in$^4$

Top, Tension: $8 = -\frac{(-10P)(4.25)}{37.25}$

Top, Comp.: $-18 = -\frac{(20 P)(4.25)}{37.25}$

Bot. Tension: $8 = -\frac{(20 P)(-1.75)}{37.25}$

Bot. Comp.: $-18 = -\frac{(-10 P)(-1.75)}{37.25}$

$\text{Smallest value of P is the allowable value} \quad P = 7.01 \text{ kips}$
6.3 A square box beam is made of two 20 × 80-mm planks and two 20 × 120-mm planks nailed together as shown. Knowing that the spacing between the nails is s = 50 mm and that the allowable shearing force in each nail is 300 N, determine (a) the largest allowable vertical shear in the beam, (b) the corresponding maximum shearing stress in the beam.

\[
I = \frac{1}{12} b h^3 - \frac{1}{12} b h^3
\]
\[
= \frac{1}{12} (120)(120)^3 - \frac{1}{12} (80)(80)^3 = 13.8667 \times 10^6 \text{ mm}^4
\]
\[
= 13.8667 \times 10^{-6} \text{ m}^4
\]

(a) \[A_1 = (120)(20) = 2400 \text{ mm}^2\]
\[\bar{y}_1 = 50 \text{ mm}\]
\[Q_1 = A_1 \bar{y}_1 = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-3} \text{ m}^3\]
\[q_{\text{max}} = \frac{2 F_{\text{max}}}{b} = \frac{(2)(300)}{50 \times 10^{-3}} = 12 \times 10^3 \text{ N}\]
\[q = \frac{VQ}{I}
\]
\[V = \frac{q I}{Q} = \frac{(12 \times 10^3)(13.8667 \times 10^{-6})}{120 \times 10^{-3}}
\]
\[= 1.3867 \times 10^3 \text{ kN} = 1.387 \text{ kN}\]

(b) \[Q = Q_1 + (2)(20)(40)(20)
\]
\[= 120 \times 10^3 + 32 \times 10^3 = 152 \times 10^3 \text{ mm}^3
\]
\[= 152 \times 10^{-3} \text{ m}^3
\]
\[\tau_{\text{max}} = \frac{VQ}{I t} = \frac{(1.38667 \times 10^3)(152 \times 10^{-6})}{(13.8667 \times 10^{-6})(2 \times 20 \times 10^3)}
\]
\[= 380 \times 10^3 \text{ Pa} = 380 \text{ kPa}\]
Problem 6.10

6.9 through 6.12 For the beam and loading shown, consider section n-n to determine (a) the largest shearing stress in that section, (b) the shearing stress at point a.

(a) \[ I = I_1 + 4I_2 = \frac{1}{12}b_1h_1^3 + 4\left(\frac{1}{12}b_2h_2^3 + A_2d_2^2\right) = \frac{1}{12}(100)(150)^3 + 4\left[\frac{1}{12}(50)(12)^3 + (50)(12)(69)^2\right] = 28.125 \times 10^6 + 4\left[0.0072 \times 10^6 + 2.8566 \times 10^6\right] = 39.58 \times 10^6 \text{ mm}^4 = 3.958 \times 10^{-6} \text{ m}^4 \]

\[ Q = A_1\bar{y}_1 + 2A_2\bar{y}_2 = (100)(75)(87.5) + (2)(50)(12)(69) = 364.05 \times 10^3 \text{ mm}^3 = 364.05 \times 10^{-6} \text{ m}^3 \]

\[ \tau_{max} = \frac{VQ}{It} = \frac{(10 \times 10^3)(364.05 \times 10^{-6})}{(3.958 \times 10^{-6})(0.100)} = 920 \times 10^2 \text{ Pa} = 920 \text{ kPa} \]

(b) \[ Q = A_1\bar{y}_1 + 2A_2\bar{y}_2 = (100)(40)(55) + (2)(50)(12)(69) = 302.8 \times 10^3 \text{ mm}^3 = 302.8 \times 10^{-6} \text{ m}^3 \]

\[ \tau = \frac{VQ}{It} = \frac{(10 \times 10^3)(302.8 \times 10^{-6})}{(3.958 \times 10^{-6})(0.100)} = 765 \times 10^3 \text{ Pa} = 765 \text{ kPa} \]
Problem 6.15

6.15 For the beam and loading shown, determine the minimum required depth \( h \), knowing that for the grade of timber used, \( f_{cu} = 1750 \) psi and \( f_{ct} = 130 \) psi.

Total load \( (750 \text{ lb/ft})(16 \text{ ft}) = 12 \times 10^3 \text{ lb} \)

Reaction at A \( R_A = 6 \times 10^3 \text{ lb} \)

\[ V_{\text{max}} = 6 \times 10^3 \text{ lb} \]

\[ M_{\text{max}} = \frac{1}{2} (8 \text{ ft})(6 \times 10^3) = 24 \times 10^3 \text{ lb-ft} = 288 \times 10^3 \text{ lb-in} \]

Bending \( S = \frac{6}{b}bh^2 \) for rectangular section.

\[ S = \frac{M_{\text{max}}}{b} = \frac{288 \times 10^3}{1750} = 164.57 \text{ in}^3 \]

\[ h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{6(164.57)}{5}} = 14.05 \text{ in} \]

Shear \( I = \frac{1}{12}bh^3 \) for rectangular section.

\[ A = \frac{1}{2}bh \]

\[ \bar{y} = \frac{1}{4}h \]

\[ Q = A\bar{y} = \frac{1}{2}bh^2 \]

\[ \tau_{\text{max}} = \frac{VQ}{Ib} = \frac{3V_{\text{max}}}{2bh} \]

\[ h, = \frac{3V_{\text{max}}}{2bh} = \frac{(3)(6 \times 10^3)}{(8)(5)(130)} = 13.85 \text{ in.} \]

The larger value of \( h \) is the minimum required depth. \( h = 14.05 \text{ in.} \).