**NODES LINE**

**Synopsis:**
The **NODES LINE** command is used to specify nodes along a straight line in the plane or in space.

**Syntax:**
```
NODES LINE NUMBER ## X1 ## X2 ## ( X3 ## )
INCREMENT ## RATIO ##
(X1_extra ##) (X2_extra ##) (X3_extra ##)
```

**Explanatory Remarks:**
By specifying values for \( X1_{\text{EXTRA}} \), \( X2_{\text{EXTRA}} \) and/or \( X3_{\text{EXTRA}} \), the straight line is mapped to a curve by means of a quadratic interpolation. The default \( X1 \), \( X2 \) and \( X3 \) coordinates are zero (0.0); as such if a specific coordinate is equal to zero, the analyst need not supply the value explicitly. The default \( \text{INCREMENT} \) value is zero (0), and the default \( \text{RATIO} \) is one (1.0). The default \( X1_{\text{EXTRA}} \), \( X2_{\text{EXTRA}} \) and \( X3_{\text{EXTRA}} \) coordinates are zero (0.0).

If the location of a node is prescribed more than once in the input and the coordinates are not in agreement, the last description is used. If, however, in a second (or subsequent description) the node number is entered as negative, then the previous location associated with this node is assumed.

Whenever several sequential node points lie along a straight line or along a curve, the coordinate generation option associated with the **NODES LINE** command, with either linear or quadratic interpolation may be used. If such a situation exists, it is only necessary to enter the coordinates of the initial and final points of the sequence (denoted in this discussion by \( N \) and \( N' \), respectively), and the values of \( \text{INCREMENT} \), \( \text{RATIO} \), and possibly \( X1_{\text{EXTRA}} \), \( X2_{\text{EXTRA}} \) and \( X3_{\text{EXTRA}} \). The constant \( \text{INCREMENT} \) represents the difference between any two successive node numbers in...
the sequence and \texttt{RATIO} defines the ratio of the distances between any two adjacent pairs of node points. The values specified in conjunction with the \texttt{X1\_EXTRA}, \texttt{X2\_EXTRA} and \texttt{X3\_EXTRA} keywords determine whether linear or quadratic interpolation will be used.

- \textbf{Linear Interpolation}

This generation option is realized if neither of the keywords \texttt{X1\_EXTRA}, \texttt{X2\_EXTRA} and \texttt{X3\_EXTRA} are specified in a \texttt{NODES\ LINE} command. If, for the input record describing node \( N' \), the value specified in conjunction with the \texttt{INCREMENT} keyword is nonzero, then intermediate node points will be generated along a straight line between node \( N' \) and the point \( N \) described in the preceding node specification record. That is, the coordinates of the nodes \((N + \texttt{INCREMENT}), (N + 2\times\texttt{INCREMENT}), \ldots, (N' - \texttt{INCREMENT})\) will each be automatically generated (see figure below).

![Figure 4. Nodal Generation Using Linear Interpolation](image)

The node number \( N' \), for which the specified non-zero value of \texttt{INCREMENT} triggered the generation of the line \( N \rightarrow N' \), can also serve as the initial point of a line generated between it and the point described in the \textit{following} record. The exterior nodes shown below can thus be generated with the following five records:

\begin{verbatim}
  nodes line number  1    x1  1.25  x2  0.20
  nodes line number  6    x1  7.50  x2  0.20  incr  1
  nodes line number 42    x1  7.50  x2  8.10  incr  6
  nodes line number 37    x1  1.25  x2  8.10  incr -1
  nodes line number -1                        incr -6
\end{verbatim}
Figure 5. Hypothetical Finite Element Mesh

Since in the second specification of node 1 the node number is entered as negative, the coordinates need not be repeated – the previous location of (1.25, 0.20) is used.

The end points of a line sequence may be entered in either order. For example, the segments illustrated below can be defined by specifying the nodes in either the order from 7 to 22; i.e.,

\[
\begin{align*}
\text{node line number 7} & \quad x1 \ 16.5 \quad x2 \ 0.50 \\
\text{node line number 22} & \quad x1 \ 7.2 \quad x2 \ 10.8 \quad \text{incr} \ 5 \quad \text{ratio} \ 2.0
\end{align*}
\]

or in the order from 22 to 7; i.e.,

\[
\begin{align*}
\text{node line number 22} & \quad x1 \ 7.2 \quad x2 \ 0.8 \\
\text{node line number 7} & \quad x1 \ 16.5 \quad x2 \ 0.50 \quad \text{incr} \ -5 \quad \text{ratio} \ 0.50
\end{align*}
\]
Figure 6. Example of Linear Nodal Coordinate Generation

The spacing of the intermediate points (e.g., nodes 12 and 17 in the above example) is controlled by the spacing RATIO. A value of 1.0 associated with the RATIO keyword results in equally spaced nodes. In selecting a spacing RATIO value, the following three formulas are of use (in the sequel the value specified in conjunction with the RATIO keyword is denoted by $r$, and the number of segments comprising a sequence of nodes is denoted by $m$):

1) If the total length of a line of nodes is equal to $L$ and if $\text{RATIO} \neq 1.0$, then the length of the first segment will equal

$$\frac{L(1 - \text{RATIO})}{(1 - \text{RATIO}^m)}$$

To better illustrate the use of this equation, consider the example shown above. Instead of the nodal spacing shown, assume that the first segment (from node 7 to 12) should be one quarter of the overall length $L$. As such, it follows that since $m = 3$ that

$$\frac{L(1 - \text{RATIO})}{(1 - \text{RATIO}^3)} = \frac{L}{4} \Rightarrow \frac{(1 - \text{RATIO})}{(1 - \text{RATIO}^3)} = \frac{1}{4} \Rightarrow \text{RATIO}^3 - 4(\text{RATIO}) + 3 = 0$$

Recalling that $\text{RATIO} \neq 1.0$, the only practical root of this cubic is found to be $\text{RATIO} = 1.303$. 
2) If the ratio of the lengths of the last and the first segments must equal a specified value \(D\) (i.e.; \(D = \Delta L_m/\Delta L_1\)), then specify a value of RATIO equal to the quantity \(D^r\), where \(r = (m - 1)^{-1}\) and where \(m > 1\). For the example shown above, if the ratio of the last segment to the first is desired to be equal to 1.50, then since \(r = (m - 1)^{-1} = (3 - 1)^{-1} = 0.50\), it follows that \(\text{RATIO} = (1.50)^{0.50} = 1.225\).

3) Suppose a line of nodes has been generated with a particular value of RATIO and that this line is to now be regenerated with twice as many segments (i.e.; with twice as many node points minus one). Furthermore suppose that it is desired to have the locations of the first set of nodes be retained in the second set. To achieve the above-mentioned goals use \( \sqrt{\text{RATIO}} \) as the new spacing ratio.

• Quadratic Interpolation

In this option quadratic isoparametric generation is employed, and facilitates the placement of node points along curved lines. The option is realized by specifying either of the keywords X1_EXTRA, X2_EXTRA, or X3_EXTRA. The values associated with these keywords represent the \(x_1\)-, \(x_2\)- and \(x_3\)-coordinates of an intermediate point through which the line of nodes will pass. In general, this intermediate point does not coincide with any node point.

For the case of quadratic interpolation, the location of a node point is determined from the following mapping (refer to the figure shown below which, for simplicity, is restricted only to two-dimensional space):

\[
\begin{align*}
    x_1(\xi_p) &= \frac{1}{2} \xi_p (\xi_p - 1) x_{1N} + (1 - \xi_p^2) x_{1e} + \frac{1}{2} \xi_p (\xi_p + 1) x_{1N}' \\
    x_2(\xi_p) &= \frac{1}{2} \eta_p (\eta_p - 1) x_{2N} + (1 - \eta_p^2) x_{2e} + \frac{1}{2} \eta_p (\eta_p + 1) x_{2N}' \\
    x_3(\xi_p) &= \frac{1}{2} \zeta_p (\zeta_p - 1) x_{3N} + (1 - \zeta_p^2) x_{3e} + \frac{1}{2} \zeta_p (\zeta_p + 1) x_{3N}'
\end{align*}
\]

where

\((x_{1N}, x_{2N}, x_{3N}) = \text{coordinates of the initial node in the sequence;}\)

\((x_{1e}, x_{2e}, x_{3e}) = \text{coordinates of the intermediate point (not necessarily a node) in the sequence;}\)

\((x_{1N}', x_{2N}', x_{3N}') = \text{coordinates of the final node in the sequence;}\)
ξ, η = local (natural) coordinates (−1 ≤ ξ, η ≤ 1) defined by

\[ \xi = \frac{x - 0.5(x_{1N} + x_{1N'})}{\frac{1}{2}(x_{1N} - x_{1N'})} \], \quad \eta = \frac{y - 0.5(x_{2N} + x_{2N'})}{\frac{1}{2}(x_{2N} - x_{2N'})} \]

and,

\((\xi_p, \eta_p)\) = coordinates of node “p” in (ξ, η) space.

The following example serves to better compare and contrast the linear and quadratic interpolation options. Consider the generation from node 5 (2.50, 0.80, 0.0) to node 25 (22.50, 13.14, 0.0). Using linear interpolation with spacing RATIOS of 1.0 and 0.75, the coordinates shown below are generated.
Figure 8. Example of Nodal Coordinate Generation Using Linear Interpolation

Quadratic interpolation is next employed, with the “extra” (intermediate) point chosen to have the coordinates (16.0, 13.5, 0.0). The nodal coordinates generated using RATIOS of 1.0 and 0.75 are shown below.
Example:

The simplest application of the NODES LINE command is to specify the location of a single node. For example, if the coordinates of node 536 are (12.83, −5.283, 0.54), enter the following command:

```
nodes line number 536 x1 12.83 x2 -5.283 x3 0.54
```
Using the following set of commands, the *boundary* of the nine-node region shown below is described.

```
nod lin num 1
nod lin num 3  x1 10.0           inc  1
nod lin num 9  x1 10.0  x2 10.0  inc  3
nod lin num 7           x2 10.0  inc -1
nod lin num -1                    inc -3
```

Note that advantage has been taken of the fact that the default coordinate values are equal to zero.

**See also:**

The *ELEMENTS*, *GENERATE SURFACES*, *NODES CIRCLE* and *NODES FROM* commands.

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*Figure 10. Example Finite Element Mesh*