ELEMENT MIXED Q8P3d commands

Synopsis

The ELEMENT MIXED Q8P3d command is used to describe all mixed, 8-node quadratic quadrilateral continuum elements with a linear discontinuous pressure approximation.

Remarks

- The Q8P3d is a mixed, quadrilateral isoparametric “Serendipity” continuum element [3]. The element
  - Contains four (4) vertex nodes.
  - Contains four (4) mid-side nodes.
  - Has two (2) displacements degrees of freedom at each node, for a total of sixteen (16) displacement degrees of freedom.
  - Employs a quadratic approximation for the displacement field.
  - Employs a linear, discontinuous approximation for the pressure (Figure 1). The pressure degrees of freedom are “condensed” out [3] from the element prior to its assembly into the global system of equations.

- Since mixed or mixed-penalty continuum elements are typically used in simulating incompressible or nearly incompressible material response, Q8P3d elements should be either plane strain or torsionless axisymmetry analyses. No benefits are realized by using Q8P3d elements for plane stress idealizations. Indeed, more accurate results are obtained by using the Q8P0 element, which is the irreducible counterpart to the Q8P3d element.

- For analyses involving standard (compressible) materials no benefit is gained from using Q8P3d elements. In such cases the irreducible Q8P0 element is preferred.

- The Q8P3d element does not satisfy the Babuška-Brezzi condition and is considered to be a “borderline” element [2]. Consequently, the use of the bi-quadratic Q9P3d element is generally preferred. The Q8P3d element is, however, included in the APES computer program to facilitate comparisons with the Q9P3d element, as well as other mixed-penalty elements.
Syntax

The following syntax is used to describe a typical mixed or mixed-penalty Q8P3d continuum element:

```
ELEment MIXed TYPe [ Q8P3d ] NODes #:#:#
   (MATERIAL #) (INIitial #)
   (CONstruction #) (EXCavation #) (THIckness #:.#) (PENalty)
   (1_Additional #) (1_Increment #)
   (2_Additional #) (2_Increment #)
   (DONT_PRINT_Results)
   (DONT_PRINT_СтRAins) (DONT_PRINT_STREsses)
   (PRINT_AVG_STRAins) (PRINT_AVG_STREsses)
   (PRINT_PRIN_STRAins) (PRINT_PRIN_STREsses)
   (PRINT_VOLumetric_strain) (PRINT_AVG_VOLumetric_strain)
   (PRINT_AVG_PREssure)
```

Explanatory Notes

- The numbering order of NODES associated with Q8P3d elements, which must be specified sequentially from 1 to 8, is shown in Figure 1.

![Figure 1: Typical Mixed Q8P3d Element (discontinuous pressure field)](image)

- The MATERIAL keyword is used to specify the number of the material idealization associated with the element. The default values for the MATERIAL number is one (1).
• The **INITIAL** keyword is used to specify the initial state number associated with the element. The *default* value for the **INITIAL** is zero (0).

• The **THICKNESS** keyword is used to specify the material thickness assumed for the element. Over a given element, the thickness is assumed to be constant. The *default* **THICKNESS** value is equal to one (1.0). For **AXISYMMETRIC** and **PLANE STRAIN** idealizations (see discussion of the **ANALYSIS IDEALIZATION** command), the **THICKNESS** must be equal to 1.0. For such idealizations, specified values different from 1.0 are ignored and the proper value is used.

• The incremental **CONSTRUCTION** and **EXCAVATION** numbers represent the time increment in which the material in this element(s) is added to or removed from the model. A **CONSTRUCTION** number equal to zero corresponds to a material in existence at the beginning of the analysis. Since this is the *default* condition, no input is required in such a case. The condition of no excavation is likewise the default.

• If the **PENALTY** keyword is specified, the mixed/penalty version of the element is instead adopted in lieu of a traditional mixed version. If the mixed element is to be used to simulate relatively compressible response (e.g., Poisson’s ratio in the range from 0.0 to 0.40), then the mixed version of the element should be used. As the incompressible limit is approached, both the mixed and mixed/penalty formulations give essentially *identical* results.

• The purpose of the **PRINT** commands is to eliminate unnecessary output generated by APES. More precisely, if the time history of strains and/or stresses is desired only for a select few elements, this option greatly speeds program output and facilitates inspection of results by the user. Information associated with the elements specified in this section will be printed for every solution (time) step. If generation is performed using this **ELEMENT MIXED** command, then all the elements generated will be affected in a like manner by the above print control commands.

• Specification of the keyword **DONT_PRINT_Results** indicates that the analyst does not desire to see output of secondary dependent variables (i.e., strains and stresses) for this element.

• Specification of the **DONT_PRINT_STRAINS** keyword indicates that element strains are not to be printed. Under the *default* condition both strains are printed.

• Specification of the keyword **DONT_PRINT_STRESSES** indicates that stresses are not to be printed. Under the *default* condition stresses are printed.

• The **PRINT_PRIN_STRAINS** keyword indicates that principal strains are to be computed and printed for the element. Under the *default* condition these quantities are not computed and printed.

• The **PRINT_PRIN_STRESSES** keyword indicates that principal stresses are to be computed and printed for the element. Under the *default* condition these quantities are not computed and printed.

• The **PRINT_AVG_STRAINS** keyword indicates that average strains (averaged over the secondary quadrature points) are to be computed and printed for the element. Under the *default* condition average strains are not computed and printed.
• The **PRINT_AVG_STRESSES** keyword indicates that average stresses (averaged over the secondary quadrature points) are to be computed and printed for the element. Under the default condition average stresses are not computed and printed.

• The keyword **PRINT_VOLUMETRIC_STRAIN** causes the volumetric strain to be computed and printed for the element. In addition, the ratio of the absolute value of the volumetric strain to the absolute value of the minimum non-zero normal strain in the element is printed. That is,

\[
\frac{|\varepsilon_{vol}|}{\min(\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33})}; \min(\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}) \neq 0
\]

This ratio is instructive in the assessment of mixed and mixed/penalty elements used to simulate material response in the incompressible limit. Under the default condition the volumetric strain and the aforementioned ratio are not computed and printed.

• The keyword **PRINT_AVG_VOLUMETRIC_STRAIN** causes the average volumetric strain to be computed and printed for the element. In addition, the ratio of the absolute value of the average volumetric strain to the absolute value of the minimum average non-zero normal strain in the element is printed. This ratio is also instructive in the assessment of mixed and mixed/penalty elements used to simulate material response in the incompressible limit. Under the default condition the average volumetric strain and the aforementioned ratio are not computed and printed.

• The keyword **PRINT_AVG_PRESSURE** causes the average pressure to be computed and printed for the element. This value represents the average of the approximate pressures at the three interior pressure nodes.
Example of Command Usage

Simulation of Incompressible Material Response under Plane Strain Conditions

Consider a simple four-element mesh of Q8P3d elements. Although the solution domain is square with a side dimension of 2.0, the middle node is purposely not placed at the centroid of the solution domain Ω. As such, the elements are mildly distorted.

A distributed traction of 20.0, acting in the negative $x_2$-coordinate direction, is applied along the top boundary of Ω. Along the right boundary, a distributed traction of 10.0, acting in the negative $x_1$-coordinate direction, is applied.

The material is characterized using the isotropic constitutive model valid for both compressible and incompressible elasticity [1]. It is assumed to be essentially incompressible, with a Poisson’s ratio ($\nu$) equal to 0.499999. The elastic modulus ($E$) is taken equal to 7.50. Consequently, the Lamé parameters are

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} = 1.250 \times 10^6$$

and

$$\mu = \frac{E}{2(1+\nu)} = 2.50$$

The resulting displacements along the right boundary of Ω are equal to 2.000. Along the top boundary, the displacement is equal to -2.000.

The associated strains are $\varepsilon_{11} = 1.000$ and $\varepsilon_{22} = -1.000$, with $\varepsilon_{33} = 0$ by assumption of plane strain conditions. In addition, the engineering shear strain is a numeric zero (i.e., $\gamma_{12} \approx 0$).

Since $\sigma_{11} = -10.0$ and $\sigma_{22} = -20.0$ as a result of the specified distributed tractions, it follows that under plane strain conditions

$$\sigma_{33} = \nu(\sigma_{11} + \sigma_{22}) = -15.0$$

Finally, in the incompressible limit the pressure associated with the isotropic elastic material idealization approaches the mean pressure $\bar{p}$; viz.,

$$\bar{p} = \frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33}) = -15.0$$

The input data associated with this problem is given next.

```
ana title "simple 'patch' to test element behavior in the incompressible limit"
ana title "2 x 2 mesh of Q8P3d mixed quadrilaterals"
!
analysis action analyze
analysis type mechanical
analysis idealization plane_strain
analysis temp transient
!
echo func off
echo grav off
```
Using the above data in conjunction with the APES computer program, the results shown below are obtained. For clarity, the “header” that is printed at the top of the file is omitted from this file.

simple ‘patch’ to test element behavior in the incompressible limit 2 x 2 mesh of Q9P3d mixed quadrilaterals
Largest NODE number which can used in the mesh = 25
Max. no. of INCOMPRESSIBLE LINEAR ELASTIC materials = 1
Max. no. of 9-node quad. mixed (Q9P3d) elements = 4

-- MECHANICAL analysis shall be performed
-- Fluid flow is NOT accounted for in the analysis
-- Thermal effects are NOT accounted for in analysis

-- TWO-DIMENSIONAL solution domain assumed
   (PLANE STRAIN idealization)

-- Nodal coordinates will NOT be updated

-- solver type used : SKYLINE
-- storage type : SYMMETRIC

-- "Isoparametric" mesh generation scheme used

In approximating time derivatives, the value of "THETA" = 5.000E-01
---> LINEAR analysis

============================================================================
= M A T E R I A L  I D E A L I Z A T I O N S  =
============================================================================

---> Material number:  1

 type : incompressible (or nearly so) linear elastic
 info. : a Herrmann type material idealization

Elastic shear modulus = 2.500E+00

Elastic bulk modulus of the solid phase = 0.000E+00
Material density of the solid phase = 0.000E+00
Combined bulk modulus for solid/fluid = 1.250E+06

============================================================================
= N O D A L  C O O R D I N A T E S  =
============================================================================

node :  1  x1 = -5.000E-22  x2 = -5.000E-22
node :  2  x1 = 5.000E-01  x2 = 1.000E-41
node :  3  x1 = 1.000E+00  x2 = 1.000E-21
node :  4  x1 = 1.500E+00  x2 = 1.076E-37
node :  5  x1 = 2.000E+00  x2 = -5.000E-22
node :  6  x1 = -1.073E-37  x2 = 5.000E-01
node :  7  x1 = 5.500E-01  x2 = 5.500E-01
node :  8  x1 = 1.100E+00  x2 = 6.000E-01
node :  9  x1 = 1.550E+00  x2 = 5.500E-01
node : 10  x1 = 2.000E+00  x2 = 5.000E-01
node : 11  x1 = 1.000E-21  x2 = 1.000E+00
node : 12  x1 = 6.000E-01  x2 = 1.100E+00
node : 13  x1 = 1.200E+00  x2 = 1.200E+00
node : 14  x1 = 1.600E+00  x2 = 1.100E+00
node : 15  x1 = 2.000E+00  x2 = 1.000E+00
node : 16  x1 = 5.999E-36  x2 = 1.500E+00
node : 17  x1 = 5.500E-01  x2 = 1.550E+00
node : 18  x1 = 1.100E+00  x2 = 1.600E+00
node : 19  x1 = 1.550E+00  x2 = 1.550E+00
node : 20  x1 = 2.000E+00  x2 = 1.500E+00
node : 21  x1 = -5.000E-22  x2 = 2.000E+00
node : 22  x1 = 5.000E-01  x2 = 2.000E+00
node : 23  x1 = 1.000E+00  x2 = 2.000E+00
node : 24  x1 = 1.500E+00  x2 = 2.000E+00
node : 25  x1 = 2.000E+00  x2 = 2.000E+00

======================================================================
= ELEMENT INFORMATION =
======================================================================

--> number: 1 (type: Q9P3d ) (kind: MIXED )
-------
  nodes:  1  3  13  11  2  8  12  6  7
integration rule for primary variables: 3 x 3 Gauss-Legendre
integration rule for secondary variables: 2 x 2 Gauss-Legendre
material no. = 1
material type: incompressible (or nearly so) linear elastic
thickness = 1.000E+00

---------------------------------------------------------------------

--> number: 2 (type: Q9P3d ) (kind: MIXED )
-------
  nodes:  3  5  15  13  4  10  14  8  9
integration rule for primary variables: 3 x 3 Gauss-Legendre
integration rule for secondary variables: 2 x 2 Gauss-Legendre
material no. = 1
material type: incompressible (or nearly so) linear elastic
thickness = 1.000E+00

---------------------------------------------------------------------

--> number: 3 (type: Q9P3d ) (kind: MIXED )
-------
  nodes: 11 13 23 21 12 18 22 16 17
integration rule for primary variables: 3 x 3 Gauss-Legendre
integration rule for secondary variables: 2 x 2 Gauss-Legendre
material no. = 1
material type: incompressible (or nearly so) linear elastic
thickness = 1.000E+00

---------------------------------------------------------------------

--> number: 4 (type: Q9P3d ) (kind: MIXED )
-------
  nodes: 13 15 25 23 14 20 24 18 19
integration rule for primary variables: 3 x 3 Gauss-Legendre
integration rule for secondary variables: 2 x 2 Gauss-Legendre
material no. = 1
material type: incompressible (or nearly so) linear elastic
thickness = 1.000E+00

---------------------------------------------------------------------

======================================================================
= NODE POINT SPECIFICATIONS =
======================================================================
### Node Specification

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(-5.000E-22, -5.000E-22)</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>(5.000E-01, 1.000E-41)</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>(1.000E+00, 1.000E-21)</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
</tr>
<tr>
<td>4</td>
<td>(1.500E+00, 1.076E-37)</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
</tr>
<tr>
<td>5</td>
<td>(2.000E+00, -5.000E-22)</td>
<td>-1.667E+00</td>
<td>0</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
</tr>
<tr>
<td>6</td>
<td>(-1.073E-37, 5.000E-01)</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
</tr>
<tr>
<td>10</td>
<td>(2.000E+00, 5.000E-01)</td>
<td>-6.667E+00</td>
<td>0</td>
<td>0.000E+00</td>
<td>0</td>
<td>0.000E+00</td>
<td>0</td>
<td>0.000E+00</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>(1.000E-21, 1.000E+00)</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
</tr>
<tr>
<td>15</td>
<td>(2.000E+00, 1.000E+00)</td>
<td>-3.333E+00</td>
<td>0</td>
<td>0.000E+00</td>
<td>0</td>
<td>0.000E+00</td>
<td>0</td>
<td>0.000E+00</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>(5.999E-36, 1.500E+00)</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
<td>0.000E+00</td>
<td>-2</td>
</tr>
<tr>
<td>20</td>
<td>(2.000E+00, 1.500E+00)</td>
<td>-6.667E+00</td>
<td>0</td>
<td>0.000E+00</td>
<td>0</td>
<td>0.000E+00</td>
<td>0</td>
<td>0.000E+00</td>
<td>0</td>
</tr>
</tbody>
</table>
21 : ( x1 = -5.000E-22, x2 = 2.000E+00 )
    displacement-1 = 0.000E+00 ; history no. = -2
    force-2 = -3.333E+00 ; history no. = 0

22 : ( x1 = 5.000E-01, x2 = 2.000E+00 )
    force-1 = 0.000E+00 ; history no. = -2
    force-2 = -1.333E+01 ; history no. = 0

23 : ( x1 = 1.000E+00, x2 = 2.000E+00 )
    force-1 = 0.000E+00 ; history no. = -2
    force-2 = -6.667E+00 ; history no. = 0

24 : ( x1 = 1.500E+00, x2 = 2.000E+00 )
    force-1 = 0.000E+00 ; history no. = -2
    force-2 = -1.333E+01 ; history no. = 0

25 : ( x1 = 2.000E+00, x2 = 2.000E+00 )
    force-1 = -1.667E+00 ; history no. = 0
    force-2 = -3.333E+00 ; history no. = 0

~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
end of mathematical model data
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At time 1.000E+00 (step no. 1) NO iteration was required

======================================================================
ELEMENT STRAINS & STRESSES =
======================================================================

--> element 1 ( type = Q9P3d ):

@ (x1 = 2.203E-01, x2 = 2.203E-01):
  eps_11 = 1.000E+00 ; eps_22 = -1.000E+00 ; eps_33 = 0.000E+00 ; gam_12 = 4.144E-08
  sig_11 = -1.000E+01 ; sig_22 = -2.000E+01 ; sig_33 = -1.500E+01 ; sig_12 = 1.036E-07

@ (x1 = 8.220E-01, x2 = 2.447E-01):
  eps_11 = 1.000E+00 ; eps_22 = -1.000E+00 ; eps_33 = 0.000E+00 ; gam_12 = 4.144E-08
  sig_11 = -1.000E+01 ; sig_22 = -2.000E+01 ; sig_33 = -1.500E+01 ; sig_12 = 1.036E-07
\begin{verbatim}
@ (x1 = 9.131E-01, x2 = 9.131E-01):
eps_{11} = 1.000E+00 ; eps_{22} = -1.000E+00 ; eps_{33} = 0.000E+00 ; gam_{12} = 2.230E-08
sig_{11} = -1.000E+01 ; sig_{22} = -2.000E+01 ; sig_{33} = -1.500E+01 ; sig_{12} = 5.576E-08

@ (x1 = 2.447E-01, x2 = 8.220E-01):
eps_{11} = 1.000E+00 ; eps_{22} = -1.000E+00 ; eps_{33} = 0.000E+00 ; gam_{12} = 4.023E-08
sig_{11} = -1.000E+01 ; sig_{22} = -2.000E+01 ; sig_{33} = -1.500E+01 ; sig_{12} = 1.006E-07
p_{1} = 1.500E+01 ; p_{2} = 1.500E+01 ; p_{3} = 1.500E+01

--> element 2 ( type = Q9P3d ):
.................................

@ (x1 = 1.245E+00, x2 = 2.447E-01):
eps_{11} = 1.000E+00 ; eps_{22} = -1.000E+00 ; eps_{33} = 0.000E+00 ; gam_{12} = 1.525E-09
sig_{11} = -1.000E+01 ; sig_{22} = -2.000E+01 ; sig_{33} = -1.500E+01 ; sig_{12} = 3.813E-09

@ (x1 = 1.798E+00, x2 = 2.203E-01):
eps_{11} = 1.000E+00 ; eps_{22} = -1.000E+00 ; eps_{33} = 0.000E+00 ; gam_{12} = 4.815E-08
sig_{11} = -1.000E+01 ; sig_{22} = -2.000E+01 ; sig_{33} = -1.500E+01 ; sig_{12} = 4.815E-08

@ (x1 = 1.822E+00, x2 = 8.220E-01):
eps_{11} = 1.000E+00 ; eps_{22} = -1.000E+00 ; eps_{33} = 0.000E+00 ; gam_{12} = -7.161E-08
sig_{11} = -1.000E+01 ; sig_{22} = -2.000E+01 ; sig_{33} = -1.500E+01 ; sig_{12} = -3.559E-08

@ (x1 = 1.336E+00, x2 = 9.131E-01):
eps_{11} = 1.000E+00 ; eps_{22} = -1.000E+00 ; eps_{33} = 0.000E+00 ; gam_{12} = -9.846E-08
sig_{11} = -1.000E+01 ; sig_{22} = -2.000E+01 ; sig_{33} = -1.500E+01 ; sig_{12} = -6.929E-08

--> element 3 ( type = Q9P3d ):
.................................

@ (x1 = 2.447E-01, x2 = 1.245E+00):
eps_{11} = 1.000E+00 ; eps_{22} = -1.000E+00 ; eps_{33} = 0.000E+00 ; gam_{12} = -2.772E-08
sig_{11} = -1.000E+01 ; sig_{22} = -2.000E+01 ; sig_{33} = -1.500E+01 ; sig_{12} = -6.929E-08

@ (x1 = 9.131E-01, x2 = 1.336E+00):
eps_{11} = 1.000E+00 ; eps_{22} = -1.000E+00 ; eps_{33} = 0.000E+00 ; gam_{12} = -6.929E-08
sig_{11} = -1.000E+01 ; sig_{22} = -2.000E+01 ; sig_{33} = -1.500E+01 ; sig_{12} = -6.929E-08

@ (x1 = 8.220E-01, x2 = 1.822E+00):
eps_{11} = 1.000E+00 ; eps_{22} = -1.000E+00 ; eps_{33} = 0.000E+00 ; gam_{12} = -6.929E-08
sig_{11} = -1.000E+01 ; sig_{22} = -2.000E+01 ; sig_{33} = -1.500E+01 ; sig_{12} = -6.929E-08

@ (x1 = 2.203E-01, x2 = 1.798E+00):
eps_{11} = 1.000E+00 ; eps_{22} = -1.000E+00 ; eps_{33} = 0.000E+00 ; gam_{12} = -1.966E-08
\end{verbatim}
sig_11 = -1.000E+01 ; sig_22 = -2.000E+01 ; sig_33 = -1.500E+01 ; sig_12 = 4.915E-08
p_1 = 1.500E+01 ; p_2 = 1.500E+01 ; p_3 = 1.500E+01

--> element 4 ( type = Q9P3d ):

@ (x1 = 1.336E+00, x2 = 1.336E+00):
eps_11 = 1.000E+00 ; eps_22 = -1.000E+00 ; eps_33 = 0.000E+00 ; gam_12 = 1.690E-08
sig_11 = -1.000E+01 ; sig_22 = -2.000E+01 ; sig_33 = -1.500E+01 ; sig_12 = 4.225E-08

@ (x1 = 1.822E+00, x2 = 1.245E+00):
eps_11 = 1.000E+00 ; eps_22 = -1.000E+00 ; eps_33 = 0.000E+00 ; gam_12 = 5.041E-08
sig_11 = -1.000E+01 ; sig_22 = -2.000E+01 ; sig_33 = -1.500E+01 ; sig_12 = 1.260E-07

@ (x1 = 1.798E+00, x2 = 1.798E+00):
eps_11 = 1.000E+00 ; eps_22 = -1.000E+00 ; eps_33 = 0.000E+00 ; gam_12 = 7.914E-08
sig_11 = -1.000E+01 ; sig_22 = -2.000E+01 ; sig_33 = -1.500E+01 ; sig_12 = 2.858E-08

@ (x1 = 1.245E+00, x2 = 1.822E+00):
eps_11 = 1.000E+00 ; eps_22 = -1.000E+00 ; eps_33 = 0.000E+00 ; gam_12 = 1.143E-08
sig_11 = -1.000E+01 ; sig_22 = -2.000E+01 ; sig_33 = -1.500E+01 ; sig_12 = 2.858E-08

p_1 = 1.500E+01 ; p_2 = 1.500E+01 ; p_3 = 1.500E+01

maximum values of element variables:

| max | eps_11 @ x1 = 2.203E-01, x2 = 2.203E-01 |
| max | eps_22 @ x1 = 2.203E-01, x2 = 2.203E-01 |
| max | eps_33 @ x1 = 2.203E-01, x2 = 2.203E-01 |
| max | gam_12 @ x1 = 1.822E+00, x2 = 8.220E-01 |
| max | sig_11 @ x1 = 2.203E-01, x2 = 1.798E+00 |
| max | sig_22 @ x1 = 2.203E-01, x2 = 1.336E+00 |
| max | sig_33 @ x1 = 2.203E-01, x2 = 2.447E-01 |
| max | sig_12 @ x1 = 1.790E-07, x2 = 1.822E+00 |

= NODAL QUANTITIES =

node : 1 ( x1 = -5.000E-22, x2 = -5.000E-22 )
    u_1 = -1.022E-25, u_2 = -2.044E-25
node : 2 ( x1 = 5.000E-01, x2 = 1.000E-41 )
    u_1 = 5.000E-01, u_2 = -1.170E-25

node : 3 ( x1 = 1.000E+00, x2 = 1.000E-21 )
    u_1 = 1.000E+00, u_2 = -3.072E-25

node : 4 ( x1 = 1.500E+00, x2 = 1.076E-37 )
    u_1 = 1.500E+00, u_2 = -1.087E-25

node : 5 ( x1 = 2.000E+00, x2 = -5.000E-22 )
    u_1 = 2.000E+00, u_2 = -1.775E-25

node : 6 ( x1 = -1.073E-37, x2 = 5.000E-01 )
    u_1 = -5.849E-26, u_2 = -5.000E-01

node : 7 ( x1 = 5.500E-01, x2 = 5.500E-01 )
    u_1 = 5.500E-01, u_2 = -5.500E-01

node : 8 ( x1 = 1.100E+00, x2 = 6.000E-01 )
    u_1 = 1.100E+00, u_2 = -6.000E-01

node : 9 ( x1 = 1.550E+00, x2 = 5.500E-01 )
    u_1 = 1.550E+00, u_2 = -5.500E-01

node : 10 ( x1 = 2.000E+00, x2 = 5.000E-01 )
    u_1 = 2.000E+00, u_2 = -5.000E-01

node : 11 ( x1 = 1.000E-21, x2 = 1.000E+00 )
    u_1 = -1.536E-25, u_2 = -1.000E+00

node : 12 ( x1 = 6.000E-01, x2 = 1.100E+00 )
    u_1 = 6.000E-01, u_2 = -1.100E+00

node : 13 ( x1 = 1.200E+00, x2 = 1.200E+00 )
    u_1 = 1.200E+00, u_2 = -1.200E+00

node : 14 ( x1 = 1.600E+00, x2 = 1.100E+00 )
    u_1 = 1.600E+00, u_2 = -1.100E+00

node : 15 ( x1 = 2.000E+00, x2 = 1.000E+00 )
    u_1 = 2.000E+00, u_2 = -1.000E+00

node : 16 ( x1 = 5.999E-36, x2 = 1.500E+00 )
    u_1 = -5.436E-26, u_2 = -1.500E+00

node : 17 ( x1 = 5.500E-01, x2 = 1.550E+00 )
    u_1 = 5.500E-01, u_2 = -1.550E+00
node : 18 ( x1 = 1.100E+00, x2 = 1.600E+00 )
  u_1 = 1.100E+00, u_2 = -1.600E+00

node : 19 ( x1 = 1.550E+00, x2 = 1.550E+00 )
  u_1 = 1.550E+00, u_2 = -1.550E+00

node : 20 ( x1 = 2.000E+00, x2 = 1.500E+00 )
  u_1 = 2.000E+00, u_2 = -1.500E+00

node : 21 ( x1 = -5.000E-22, x2 = 2.000E+00 )
  u_1 = -8.875E-26, u_2 = -2.000E+00

node : 22 ( x1 = 5.000E-01, x2 = 2.000E+00 )
  u_1 = 5.000E-01, u_2 = -2.000E+00

node : 23 ( x1 = 1.000E+00, x2 = 2.000E+00 )
  u_1 = 1.000E+00, u_2 = -2.000E+00

node : 24 ( x1 = 1.500E+00, x2 = 2.000E+00 )
  u_1 = 1.500E+00, u_2 = -2.000E+00

node : 25 ( x1 = 2.000E+00, x2 = 2.000E+00 )
  u_1 = 2.000E+00, u_2 = -2.000E+00

max | u_1 | = 2.000E+00 @ node 10 ( 2.000E+00, 5.000E-01)
max | u_2 | = 2.000E+00 @ node 21 (-5.000E-22, 2.000E+00)

apes -> end of analysis . . . . . . . .
References

