

# **Time-Dependent Behavior of Geosynthetic Reinforcement – A Review of Mathematical Models**

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## 1. Introduction

Over the years, a large amount of experimental work has been done to study the time dependent behavior of geosynthetic reinforcement (Kaliakin and Dechasakulsom 2001a). The majority of these studies focused on creep response, relaxation experiments being perceived as overly complex. Numerous mathematical models of the geosynthetics, possessing varying levels of sophistication, have been developed in conjunction with many of the aforementioned experimental studies. With minor exceptions, for purposes of mathematically representing typical geosynthetic reinforcement, the models have assumed uniaxial stress and strain states. This is in keeping with the observation (den Hoedt et al., 1994) that geosynthetics commonly used for reinforcement exhibit negligible lateral contraction.

Some models proposed to simulate creep and relaxation response of geosynthetics are reviewed herein. The discussion is limited to response under isothermal conditions, as very few models have been proposed that account for both thermal and mechanical response.

## 2. Overview of Time-Dependent Models for Geosynthetics

### 2.1. Models Proposed to Simulate Creep Response

The most basic models developed to simulate creep response of geosynthetics are simple, empirical, mathematical expressions. For example, the following expression has been proposed

$$\epsilon = \epsilon_0 + A \log t \quad (1)$$

where  $\epsilon$  represents the total strain,  $\epsilon_0$  and  $A$  are functions of stress, temperature and nature of the material, and  $t$  denotes time. Using this expression, Finnigan (1977), Van Leeuwen (1977) and Raumann (1981) have reported success in modeling short-term creep behavior.

In a somewhat similar fashion, Findley (1987) presented 26-year creep data for PVC and polyethylene. The data showed that the polyethylene exhibited substantially more creep than the PVC. During loading, the creep was predicted by the equation

$$\epsilon = \epsilon_0 + \epsilon_1 t^n \quad (2)$$

where  $\epsilon$  represents the strain,  $t$  is the time, and  $\epsilon_0$ ,  $\epsilon_1$  and  $n$  are constants. From other work (Findley et al. 1976), it has been shown that  $n$  is typically a constant independent of stress and temperature, whereas  $\epsilon_0$  and  $\epsilon_1$  are stress and temperature dependent. In addition, Lai and Findley (1973) found  $n$  to be generally less than one.

According to the Boltzman superposition principle (Boltzman 1876), if two or more stresses are applied sequentially (e.g., in a creep experiment), the stresses act independently and the resulting strains add linearly. Likewise, in a stress relaxation experiment, two or more strains could be applied sequentially and the resulting stresses would add linearly. As such, the strain during recovery is given by

$$\epsilon = \epsilon_0 + \epsilon_1 t^n - \left[ \epsilon_0 + \epsilon_1 (t - t_1)^n \right] = \epsilon_1 \left[ t^n - (t - t_1)^n \right] ; \quad t > t_1 \quad (3)$$

In an effort to numerically characterize geotextiles, Das (1990) modified the creep strain rate relationship proposed by Singh and Mitchell (1968). In particular

$$\dot{\epsilon} = c_1 c_2 \bar{p} \frac{t_1}{t}^m \quad (4)$$

where  $c_1$ ,  $c_2$  and  $m$  represent material constants,  $t$  denotes the time,  $t_1$  is a reference time,  $\bar{p} = \frac{P}{P_u}$  where  $P$  represents the sustained load per unit width, and  $P_u$  is the strip tensile strength. Das (1990) claimed to have tested three geotextiles (non-woven, heat bonded filaments of *polypropylene*; two non-woven, needle-punched, continuous fiber *polyester* geotextiles), but showed no results. He did, however, indicate that the values selected for  $m$  were consistent with those presented by Shresta and Bell (1982).

Based on the results of confined creep tests, Matichard et al. (1990) and Blivet et al. (1992) propose the following expression between the creep strain  $\epsilon_c$  and time  $t$

$$\epsilon_c = \epsilon_1 t^n \quad (5)$$

where  $\epsilon_1$  represents the strain at the end of the loading phase. Blivet et al. (1992) noted that for woven *polypropylene*, tested with or without confinement, the value of  $n$  is about 0.10. For woven *polyester*, the value of  $n$  is about 0.01, and is again independent of the presence of confining soil. For non-woven geotextiles, the values of  $n$  are similar to those for woven geotextiles. For *polypropylene*  $n$  is about 0.12; for *polyester* it is about 0.015. These values are in agreement with ones determined by Matichard et al. (1990).

Viezee et al. (1990) found that measured creep could be predicted by the expression

$$\epsilon_c(t) = m \log_{10} t + \epsilon_c(t_0) \quad (6)$$

where  $\epsilon_c(t_0)$  denotes the intercept at one hour (in percent),  $m$  is the creep gradient (in percent per decade), and  $t$  denotes time. They present values of  $\epsilon_c(t_0)$  and  $m$  for two polyester yarns, both damaged and undamaged. The same expression was used by Miki et al. (1990) to represent the primary and secondary phases of creep of spun-bonded non-woven fabrics.

Next in complexity are rheological models consisting of combinations of springs and dashpots. A three-element rheological model consisting of a spring and a Kelvin model in series was used by Paute and Segouin (1977) to model the creep behavior of geotextiles. The test data presented was, however, of very short duration (8 hours).

Shrestha and Bell (1982) modeled the time dependent behavior of geotextiles by a four-parameter rheological (Berger) model, and the three-parameter creep formula proposed by Singh and Mitchell (1968) for the simulation of triaxial creep of soils. In the rheological model the viscous element was represented by two constants whose values were determined from the Rate Process Theory (Eyring et al., 1941). The basic difference between the Rate Process Theory based approach and the three-parameter model is that in the former the creep rate is considered as being continuously decreasing during the

transient stage until a minimum value is reached; it remains constant at this minimum value during the secondary stage of creep until the beginning of the tertiary phase of creep; during tertiary creep, the strain rate increases very rapidly until failure. Conversely, in the three-parameter creep formula, the strain rate is considered to be continuously decreasing. Using these two approaches, Shrestha and Bell (1982) found that the creep response predicted by the four-parameter rheological model was more consistent with the experimental results. For non-woven geotextiles, the time to reach failure strains under sustained load predicted by the three-parameter model was much longer than for the four-parameter model; for woven geotextiles, both empirical methods predicted comparable times to failure. Overall, however, both methods predicted time to failure that was much shorter than the normal design life of geotextiles, even at stress levels as low as 30% of ultimate.

An extrapolation method, based on a partial rheological model has been presented by McGown et al. (1984). A related approach uses the extrapolation of isochronous stiffness and time correlation curves (Andrawes et al. 1986).

Concerning rheological models consisting of springs and dashpots, Müller-Rochholz and Kirscher (1990) point out that such models accurately predict the response of the polymers themselves. However, for geotextiles, especially nonwoven materials, such models do not always give accurate predictions of response. They attribute this to the load bearing capacity created by local friction, bending, shear and tension loading.

Another class of models, admittedly more complex than rheological models, is that based on integral techniques. For example, the multiple integral technique suggested by Onaran and Findley (1965) has been found to be useful in representing the non-linear viscoelastic behavior of a range of geotextiles and geogrids (Kabir 1984, 1988; Yeo 1985). According to this technique, for uniaxial creep at some load  $p$

$$= R(t)p + R(t)p^2 + R(t)p^3 \quad (7)$$

For constant loading of geotextiles and polymers, the kernel functions  $R$ ,  $M$  and  $N$  are expected to take on the following form

$$R(t) = \mu_1 + \mu_2 t^n \quad (8)$$

$$M(t) = \mu_2 + \mu_2 t^n \quad (9)$$

$$N(t) = \mu_3 + \mu_3 t^n \quad (10)$$

where  $\mu_1, \mu_2, \mu_3, \mu_1, \mu_2, \mu_3$  represent material functions dependent on the temperature, and  $n$  is a function of the material and may or may not be a function of temperature. Substitution of the latter three equations into the first one yields the expression

$$\epsilon(t, p) = \epsilon_0(p) + \epsilon_t(p)t^n \quad (11)$$

where

$$\epsilon_0 = \mu_1 P + \mu_2 P^2 + \mu_3 P^3 \quad (12)$$

and

$$\epsilon_t = \mu_1 P + \mu_2 P^2 + \mu_3 P^3 \quad (13)$$

The seven parameters associated with this model are determined by fitting the results of creep tests for at least three different loads (Kabir 1988).

A related approach has been proposed by Findley et al. (1976). They represented the creep behavior of nonlinear viscoelastic materials by using a series of “multiple integrals.” For uniaxial creep, the following expression was proposed for the total strain

$$\epsilon(t) = F_1 p + F_2 p^2 + F_3 p^3 \quad (14)$$

where  $p$  again denotes the applied uniaxial load, and  $F_1, F_2$  and  $F_3$  represent kernel functions. To calibrate the model, isothermal uniaxial creep tests, at three different loads  $(p_a, p_b, p_c)$ , must be performed. The strains measured in these tests, denoted by  $\epsilon_a, \epsilon_b$  and  $\epsilon_c$ , respectively, are then used to determine the kernel functions. The associated equations, which are solved simultaneously for the kernel functions, are

$$\epsilon_a(t) = F_1 P_a + F_2 P_a^2 + F_3 P_a^3 \quad (15)$$

$$\epsilon_b(t) = F_1 P_b + F_2 P_b^2 + F_3 P_b^3 \quad (16)$$

$$\epsilon_c(t) = F_1 P_c + F_2 P_c^2 + F_3 P_c^3 \quad (17)$$

To effectively use the model, the magnitude of the loading must be known a priori. Thus, if tertiary creep is to be predicted, creep tests to failure must be performed. Using the model of Findley et al. (1976), Helwany and Wu (1992) were able to simulate the creep response of a polypropylene composite, heat bonded geotextile and a polypropylene non-woven heat bonded geotextile. Stress levels used in the creep tests were not high enough to result in tertiary creep, however, so the assessment of the model was incomplete.

Sawicki, et al. (1998) had studied creep behavior of geosynthetics by using standard linear solid (SLS) model see Figure 4.1. It is shown that the SLS model described by three parameters in which served as a useful low resolution approximation for a low stress levels which exclude secondary and tertiary creep.

$$\epsilon = \epsilon_1 + \epsilon_2 \quad (18)$$

$$\epsilon_1 = \frac{\sigma}{E_1} \quad (19)$$

$$\epsilon_2 = \frac{\sigma}{E_2} + \frac{d\sigma}{dt} \quad (20)$$

It contains of two basic elements joined together, namely the spring defined its stiffness  $E_1$  and the Kelvin system, characterized by the stiffness  $E_2$  and the viscosity  $\eta$ . In the one-dimensional case the model is described by the following constitutive equation, cf. Williams (1980):

$$\frac{1}{E_1} \frac{d\sigma}{dt} + \frac{\sigma}{E_1 + E_2} = \frac{d\epsilon}{dt} + \frac{\sigma}{E_2} \quad (21)$$

where  $\sigma$  = stress (force per unit width)

$\epsilon$  = total strain

t = real time

The model parameters  $E_1$ ,  $E_2$  and  $\tau$  (material constants) can be determined experimentally. In the case of creep tests performed at constant stress ( $\sigma = \text{constant}$ ), so Eq. (21) reduced to the following from:

$$\frac{d}{dt} + \frac{E_2}{E_1} = \frac{E_1 + E_2}{E_1} \quad (22)$$

The solution to Eq. (22) is:

$$-\frac{1}{E^*} - \frac{1}{E_2} e^{-\frac{E_2}{E_1} t} = \epsilon(t) \quad (23)$$

where:  $\epsilon(t)$  = the creep function

$$E^* = \frac{E_1 E_2}{E_1 + E_2} \quad (24)$$

$E$  = delayed elastic modulus of the standard rheological model

## 2.2. Models Proposed to Simulate Relaxation Response

Compared to creep models, relatively few formulations have been proposed to simulate the relaxation response of geosynthetics.

Koerner et al. (1993) presented the following “in house” formula for stress relaxation of geomembranes.

$$\sigma(t) = c t^{-b} \quad (25)$$

where  $t$  denotes time, and  $b$  and  $c$  are constants. This type of behavior has been referred to as “physical stress relaxation,” as opposed to chemical relaxation (Debnath, 1985).

Sawicki, et al. (1998) had studied stress relaxation behavior of geosynthetics by using standard linear solid (SLS) model.

$$\frac{d}{dt} + \frac{E_1 + E_2}{E_1} = \frac{E_1 E_2}{E_1} \quad (26)$$

The solution to Eq.(26) is:

$$\sigma = (E_1 - E^*)e^{-\frac{E_2 t}{\eta}} + E^* \epsilon(t) \quad (27)$$

where:  $\epsilon(t)$  = the relaxation function.

The boundaries of this equation are in the followings;

At  $t=0$ :

$$\sigma = E_1 \epsilon \quad (28)$$

At  $t \rightarrow \infty$  :

$$\sigma = E^* \epsilon \quad (29)$$

At  $t \rightarrow \infty$  :

$$\dot{\sigma} = \frac{1}{\eta} \epsilon \quad (30)$$

Or if there is more stress relaxation than the creep, the plastic elements should be built into the structure rheological. Here, the plastic element defined by the stiffness R is shown the Figure 4.2.

$$\left(1 + \frac{E_3}{R}\right) \dot{\sigma} + \frac{d\sigma}{dt} = E_3 \dot{\epsilon} + \frac{d\epsilon}{dt} \quad (31)$$

In the case of creep, the solution is in the following;

$$\sigma = \frac{1}{E_3} + \frac{1}{R} - \frac{1}{E_3} e^{-\left(\frac{E_3}{R} t\right)} \quad (32)$$

In the case of stress relaxation, the solution is in the following;

$$\sigma = \left(1 - \frac{E_1 + E_3}{R}\right) \sigma_0 e^{-\frac{E_1 + E_3}{R} t} + \frac{E_1 E_3}{E_1 + E_3} \left(1 - \frac{E_1 + E_3}{R}\right) \quad (33)$$

$E^{**} = \frac{E_1 E_3}{E_1 + E_3}$  = delay elastic modulus of the modified model.

The boundaries of this equation are in the followings;

At  $t=0$ :

$$\sigma = \sigma_0 \left(1 - \frac{E_1 + E_3}{R}\right) + \frac{E_1 E_3}{E_1 + E_3} \left(1 - \frac{E_1 + E_3}{R}\right) \quad (34)$$

At  $t \rightarrow \infty$  :

$$\sigma = \frac{E_1 E_3}{E_1 + E_3} \left(1 - \frac{E_1 + E_3}{R}\right) \quad (35)$$

At  $t \rightarrow \infty$  :

$$\frac{1}{\sigma} = \frac{1}{E_3} + \frac{1}{E_1} \quad (36)$$

### 2.3. Models Proposed to Simulate Both Creep and Relaxation

In a recent paper, Sawicki (1998) proposed rheological models for predicting the creep or relaxation response of specific geogrids. However, the models are predicated on the a priori knowledge of the specific type of response. That is, it must be known whether the geogrid will undergo creep or relaxation response; in the course of loading, the response mode cannot change. Thus, Sawicki's models, though more general than the basic rheological models discussed above, still lack true generality.

Zhang and Moore have presented a more general model that accounts for the elastic-viscoplastic response of geosynthetics (Zhang and Moore 1997). This multi-axial model that is based on the unified theory of Bodner and Partom (1975), has been shown to realistically simulate various aspects of geosynthetic response with good agreement between numerical predictions and experimental results.

## 2.4. Concluding Remarks Concerning Modeling

As evident from the overview presented in the previous section, the mathematical modeling of the time dependent behavior of geosynthetics has typically been realized using formulations specifically designed to simulate creep response, or those specifically designed to simulate relaxation. With the exception of the Zhang and Moore (1997) model, few simple yet robust models appear to have been proposed that account for both creep and relaxation in a robust yet reasonably accurate fashion.

In addition, simple mathematical models, rheological models, and integral techniques all lack one fundamental characteristic that is necessary for their implementation into finite element computer programs. Namely, using any of the aforementioned approaches, one cannot compute a consistent incremental tangent modulus  $E_t = f / f$ .

The above shortcomings manifest themselves in the inability to perform proper finite element analyses. In particular, consider the approach used by Helwany (1992) in his analysis of a geosynthetically reinforced wall with cohesive backfill. Using an integral techniques similar to that described by equations (5) to (8) he states that:

“For each time increment  $t$ , the expected creep strains in all viscoelastic bar elements are calculated. Equivalent nodal creep forces (corresponding to the expected creep strains) are then calculate and applied at the nodal points of each viscoelastic bar element. The response of the structure is then evaluated through regular finite element procedure.”

Such an approach is deficient for two reasons: First it a priori assumes creep response for the reinforcement (a condition that has been shown by Dechasakulsom (2000) not to be true for the particular wall analyzed). Secondly, by assuming “expected” creep strains, this approach precludes a consistent finite element analysis from being performed. In such an analysis, the strains are computed as secondary dependent variables from the displacements (the primary dependent variables), and are not prescribed at the outset.

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