is reasonably valid and may be used as a basis for a preliminary analysis of the overall problem (1). The interval for the time scale is taken to be one year, over which period complete mixing of each lake is probably effected. In general, consider a lake, the volume of which is \( V \), receiving fresh water flow \( R \), from the rivers in the drainage basin and, in some cases, an inflow, \( I \), from the upstream lake. The mass rate of waste discharged from the population and industrial sources is \( W \), which may be time-variable. It is obvious that both the flows and the volume \( (R, I, V) \) are variable, but considering the time scale in question, these variations are assumed to have a minimal effect. The long-term pattern of flow and volume is characterized by constant values upon which are superimposed cyclic or random variations. A mass balance is constructed taking into account the inflow and outflow and the various sources and sinks of material. Assuming that the substance under consideration is nonconservative and decays or increases in accordance with a

<table>
<thead>
<tr>
<th>Type of input</th>
<th>Form, ( W(t) )</th>
<th>Concentration, ( c(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant ( w_o )</td>
<td>( \frac{w_o}{\alpha V} (1 - e^{-\alpha t}) )</td>
<td></td>
</tr>
<tr>
<td>Linear ( w_o + w_f )</td>
<td>( \frac{w_o}{\alpha V} (1 - e^{-\alpha t}) + \frac{w_f}{\alpha V} (1 - e^{-\alpha t} - a t) )</td>
<td></td>
</tr>
<tr>
<td>Exponential ( w_o e^{\alpha t} )</td>
<td>( \frac{w_o}{\alpha V} e^{\alpha t} (1 - e^{-\alpha t}) )</td>
<td></td>
</tr>
<tr>
<td>Limiting ( w_o (1 - e^{-\alpha t}) )</td>
<td>( \frac{w_o}{\alpha V} \left[ (1 - e^{-\alpha t}) - \frac{\alpha}{(a + \alpha)} (e^{-\alpha t} - e^{-\alpha t}) \right] )</td>
<td></td>
</tr>
</tbody>
</table>

first-order reaction, the following differential equation is developed from the mass balance:

\[
\frac{dc}{dt} = \frac{W(t)}{V} - \frac{c}{I_0} - Kc \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (1)
\]

The wastewater discharge \( (W) \) is taken as a time variable, and the flows and volume are assumed constant in accordance with the foregoing analysis. The parameter, \( I_0 \), is the detention time \( Q/V \) in which \( Q \) is the flow out of the lake, \( K \) is the first-order reaction coefficient, and \( c \) is the annual average concentration. The general solution of this equation is

\[
c = \frac{Q}{V} \int_0^t W(t) e^{\alpha t} dt + c_0 e^{-\alpha t} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2)
\]

For simplification, the term \( (K + 1/I_0) \) is replaced by \( \alpha \). For conservative substances, such as chlorides, the reaction coefficient is zero and \( \alpha \) equals the reciprocal of the detention time \( 1/I_0 \). The second term takes into account the initial condition, in which \( c_0 \) is the concentration at time equal to zero. This concentration decays exponentially in time by virtue of the flushing due to flow through the lake (18) and the reaction characteristic of the substance.
The first term in Eq. 2 is the concentration change brought about by the inputs, \( W \), which takes on various functional forms. This part of the equation may be expressed more conveniently as

\[
c = V^{-1} \int_0^t e^{-\alpha t - \tau} W(\tau) \, d\tau
\]  

This equation is referred to as a convolution integral. The concentration due to an element of wastewater input \( W(\tau) \) in time \( dt \) is equal to the product of the kernel function \( e^{-\alpha t - \tau} \), and \( W(\tau) \). The total concentration, by the principle of superposition, is equal to the sum of these individual elemental concentrations up to time, \( t \), or the integral over time, \( 0 \leq t \), of the product of the wastewater input function and the kernel (5). The functional forms of the time-variable inputs indicated in Table 1 cover a range of practical conditions. These equations indicate the effect of a wastewater input on the water quality of the lake into which the waste is discharged.

In order to evaluate the effect of inputs to upstream lakes on water quality in downstream lakes, it is necessary to consider a series of completely mixed bodies. The output from one acts as an input to the downstream lake, which in turn feeds the next. Identifying the lakes from 1 (the first upstream) to \( n \) (the most downstream) the equation for the \( n \)th lake is

\[
\frac{dc_n}{dt} + \alpha_n c_n = \frac{Q_{n-1}}{V_n} c_{n-1}
\]

The solution obtained for the \( n \)th lake due to a waste load introduced in the \( i \)th is

\[
c_n = \frac{Q_{n-1}}{V_n} e^{-\alpha_{n-i} t} \int_0^t e^{\alpha_{n-i} \tau} \, d\tau \frac{Q_{n-1}}{V_{n-1}} e^{-\alpha_{n-i-1} t} \int_0^\tau e^{\alpha_{n-i-1} \tau} \, d\tau \ldots
\]

\[
\times \frac{Q_{i-1}}{V_{i-1}} e^{-\alpha_{i-1} t} \int_0^t e^{\alpha_{i-1} \tau} \, d\tau \frac{Q_i}{V_i} e^{-\alpha_i t} \int_0^t W(t) e^{\alpha_i t} \, dt
\]

which may be expressed as

\[
c_n = \left( \frac{\alpha_{n-i} \alpha_{n-i-1} \ldots \alpha_i}{V_n} \right) e^{-\alpha_n t} \int_0^t e^{\alpha_n \tau} \, d\tau
\]

\[
\times e^{-\alpha_{n-i} t} \int_0^t e^{\alpha_{n-i} \tau} \, d\tau \ldots e^{-\alpha_{i-1} t} \int_0^t W(t) e^{\alpha_i t} \, dt
\]

in which \( \alpha_i \) is the parameter for the \( i \)th lake. As an example, the effect of a constant rate of discharge into the first lake on the water quality in the third lake is

\[
c_3 = \frac{W_3}{\alpha_2 \alpha_3} \left[ (1 - e^{-\alpha_2 t}) - \alpha_2 (e^{-\alpha_2 t} - e^{-\alpha_3 t}) \right. \\
\left. + \alpha_3 (e^{-\alpha_3 t} - e^{-\alpha_3 t}) \right]
\]

in which \( \alpha_3 = \frac{\alpha_3 \alpha_2}{(\alpha_3 - \alpha_2)(\alpha_3 - \alpha_1)} \).
which covers half the width of the U.S. The area of the basin is slightly less than 300,000 sq miles of which one-third is water and two-thirds is land. More than 5,000 cu miles of water are stored in the five lakes.

The lakes differ greatly in size. Lake Superior is the deepest and has the largest volume, which with its relatively small inflow has the largest detention or displacement time. At the other extreme, Lake Erie, which is the shallowest of the five lakes, has the lowest detention time. Table 2 is a summary of the areas, volumes, mean depths and detention times of each lake (3,21). A profile of the lakes is presented in Fig. 2. The detention time is based on the assumption that the entire volume of each lake is available for dilution and flow. This is the basis of the completely mixed system.

### TABLE 2.—LAKE PARAMETERS

<table>
<thead>
<tr>
<th>Lake</th>
<th>Drainage Areas, in square miles</th>
<th>Volume, in cubic miles</th>
<th>Mean depth, in feet</th>
<th>Mean flow, in cubic feet per second</th>
<th>Detention time, in years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superior</td>
<td>31,800</td>
<td>2,940</td>
<td>487</td>
<td>71,600</td>
<td>191</td>
</tr>
<tr>
<td>Michigan</td>
<td>22,400</td>
<td>1,170</td>
<td>276</td>
<td>55,000</td>
<td>96.1</td>
</tr>
<tr>
<td>Huron</td>
<td>23,000</td>
<td>850</td>
<td>125</td>
<td>216,400</td>
<td>22.6</td>
</tr>
<tr>
<td>Erie</td>
<td>9,920</td>
<td>113</td>
<td>85</td>
<td>201,000</td>
<td>2.6</td>
</tr>
<tr>
<td>Ontario</td>
<td>7,550</td>
<td>404</td>
<td>263</td>
<td>238,000</td>
<td>7.9</td>
</tr>
</tbody>
</table>

### TABLE 3.—VALUES OF RUNOFF, LAKE PRECIPITATION, AND EVAPORATION

<table>
<thead>
<tr>
<th>Lake</th>
<th>Runoff, in cubic feet per second per square mile</th>
<th>Precipitation on Lake Surface, in inches per year</th>
<th>Evaporation From Lake Surface, in inches per year</th>
<th>Computed</th>
<th>Reported (30)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superior</td>
<td>1.00</td>
<td>29.4</td>
<td>19.4</td>
<td>22</td>
<td>22</td>
</tr>
<tr>
<td>Michigan</td>
<td>0.85</td>
<td>31.2</td>
<td>21.5</td>
<td>25</td>
<td>26</td>
</tr>
<tr>
<td>Huron</td>
<td>1.05</td>
<td>31.2</td>
<td>31.4</td>
<td>25</td>
<td>26</td>
</tr>
<tr>
<td>Erie</td>
<td>0.79</td>
<td>34.0</td>
<td>30.5</td>
<td>25</td>
<td>26</td>
</tr>
<tr>
<td>Ontario</td>
<td>1.20</td>
<td>34.3</td>
<td>31.4</td>
<td>26</td>
<td>26</td>
</tr>
</tbody>
</table>

The degree to which the system is not completely mixed in nature would be reflected in the difference between the calculated concentrations and response times and the actual values.

For conservative substances such as chlorides, the volumetric effect is relatively insignificant by contrast to the dilution. For nonconservative material, the volumetric characteristic could be the important feature.

Hydrology of Basin.—The major components of the water balance of the lakes are the inflows and outflows, runoff, precipitation and evaporation, the magnitudes of which vary considerably from year to year. A water balance is shown in Fig. 3, which indicates the average values of all components for the period 1900–1965. A description of these components follows.

Inflows and Outflows.—Flows for the St. Mary's, St. Clair, Detroit and Niagara rivers were obtained from Lake Survey records (19). All values were obtained for a time period of 1900 to 1965 except for the Detroit River flow which was averaged over the period from 1939 to 1960. This latter value is in agreement with the average for 1900 to 1965 assuming the change in the stage-discharge relationship presented by Brun (6). The estimate of the average outflow of Lake Michigan through the Mackinac Straits was taken from Ref. 3 which is in reasonable agreement with estimates by the Lake Survey (22). The outflow for Lake Ontario was derived from records at Iroquois, Ontario (14) which were adjusted to exclude the flow from the Black River basin in New York. Average values of the Chicago diversion and the Welland Canal flows were adopted as agreed upon by the International Joint Commission on Hydrology of the Great Lakes Basin (8).

It may be noted that the Ouabitch Lac diversion into Lake Superior has not been included in the water balance shown in Fig. 3, and all flows are adjusted accordingly. Since the 5,000-cfs diversion was begun in 1941, it was not representative of the entire period. In addition, the value of 5,000 cfs is not statistically significant when compared to the magnitude and variation of flow in the St. Mary's River or the Niagara River as shown in Fig. 4.

Runoff.—All values for the total basin runoff were obtained from USGS records or Canadian source material. Since many streams and rivers are not gaged, averages of unaged drainage areas were generally estimated on the basis of nearby drainage basins. Thus, the runoff in the Lake Superior basin was computed directly from available USGS records. The value for Lake Michigan is taken from Ref. 15, for Lake Huron from Ref. 10 and for Lakes Erie and Ontario from Ref. 7. These values are in close agreement with recent estimates from the Lake Survey (10).

Lake Precipitation.—The amount of rainfall on the surface of each lake was computed from yearly averages provided by the U.S. Lake Survey (19).
for the period of 1900 to 1960. It is assumed that the shore stations from which these records were derived are adequately representative of conditions existing over the lake.

Lake Evaporation.—Using the data described in the preceding sections, the evaporation from the lake surface was computed as the difference between the total flow into the basin and the total flow out. For example, using the data

![Graph of Niagara River flow](image)

![Graph of St. Mary's River flow](image)

![Cumulative Frequency Distribution](image)

**FIG. 4.—REPRESENTATIVE RIVER FLOW DISTRIBUTIONS**

shown in Fig. 3, the evaporation from the surface of Lake Superior is computed as the difference in the total inflow (69,000 cfs + 48,200 cfs = 117,200 cfs) and the outflow (71,800 cfs), which is 45,000 cfs minus the value shown on the block diagram for evaporation. The values of runoff, lake precipitation and evaporation, as computed here and from Ref. (10), are listed in Table 3.

**Basin Population.—** In order to estimate the number of people within the drainage basin of each lake, the boundaries of each basin were delineated on

![Graph of Lake Erie Basin population](image)

![Graph of Lake Michigan Basin population](image)

**FIG. 5.—GROWTH OF BASIN POPULATION**

Large scale hydrological maps and were then transferred to geographical maps upon which county borders were shown. The 1960 population was evaluated on a county to county basis as follows: the county urban population within and outside each basin was determined; the county rural population density was then computed and applied to the area within the basin. By adding the city and rural populations within the lake basins a total basin population was obtained for 1960.

Since this computation involved over 200 counties the same procedure was not repeated to estimate the population from 1900 onward. Rather, the popu-

![Graph of U.S. Chemical Products Index](image)

**FIG. 6.—INDEX OF U.S. CHEMICAL PRODUCTS**

lation of each county was obtained from census data beginning with 1900, and then multiplied by the 1960 ratio of population of the county within the basin to total county population. Finally, these estimated county values were added to establish a total basin population. Similar methods were employed, on a provincial level, to obtain the Canadian population. The population of the city of Chicago was not included since its waste waters have been diverted to the Chicago Sanitary and Ship Canal since 1900. The population data for the Lake Erie and Michigan areas are shown in Fig. 5 with the fitted curves. The following function was found to conveniently fit the data:
\[ N = N_0 e^{at} \]  \hspace{1cm} (10)

in which \( N \) is the population at time \( t \) and \( N_0 \) the initial population taken as the base.

### Table 4—Population Curve Parameters

<table>
<thead>
<tr>
<th>Lake basin</th>
<th>( N_0 ), in millions</th>
<th>( a ), per year</th>
<th>1960 population, in millions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superior</td>
<td>0.4</td>
<td>0.004</td>
<td>0.5</td>
</tr>
<tr>
<td>Michigan</td>
<td>2.2</td>
<td>0.015</td>
<td>5.4</td>
</tr>
<tr>
<td>Huron</td>
<td>0.7</td>
<td>0.014</td>
<td>1.6</td>
</tr>
<tr>
<td>Erie</td>
<td>3.2</td>
<td>0.020</td>
<td>10.6</td>
</tr>
<tr>
<td>Ontario</td>
<td>2.4</td>
<td>0.013</td>
<td>9.3</td>
</tr>
</tbody>
</table>

That in the year 1900. The parameters for the curves of all lake basins are listed in Table 4.

### Chloride Sources

The review of previous work and survey data was recently presented by Beeton (4), including references as early as the middle of the previous century. The majority of the data available concerned the chemical constituents of the lake waters—chlorides, sodium and potassium, sulfate, calcium and total dissolved solids. The major sources of these chemicals are the industrial and municipal waste water discharges, inflow from upstream lakes, and, with respect to chloride, the quantity employed in the de-icing of highways. The chloride ion is also present as a background concentration in the natural runoff and to a minor degree, in precipitation.

**Natural Background Concentrations.**—Before the development and industrialization of the Great Lakes basin, a relatively stable equilibrium concentration of chlorides was achieved. This may be inferred from the measured chloride concentrations during the period from 1850 to 1900 which show relatively constant values in Lakes Michigan, Superior and Huron. For any one lake the natural chloride inputs from upstream lakes, tributary drainage and precipitation over the lake area were in equilibrium with the chlorides flowing out of the lake, i.e.

\[ R c_f + P c_f + I c_f = Q c_f \]  \hspace{1cm} (11)

in which \( R, P, I \) and \( Q \) refer to the flows in the basin due to runoff, precipitation on the lake area, inflow from upstream lakes and outflow, respectively. The term \( c \) refers to chloride concentrations in each of these quantities and \( c_f \) is the natural background concentration of a lake.

Eq. 11 was employed to estimate \( c_f \), in the tributary streams of each basin. From observed data (4), the natural background concentration in each lake, \( c_f \), was selected and the concentration in the precipitation, \( c_p \), was in the range of 0.1 mg per l to 0.3 mg per l (13). The values of the runoff, \( R \), the precipitation, \( P \), the inflow, \( I \) and the outflow, \( Q \), were taken directly from the water balance, as shown in Fig. 3. Substitution of these values in Eq. 11 yielded a natural concentration in the river of each basin ranging from 3 mg per l to 4 mg per l.

![Graph](Image)

**FIG. 7.—Municipal Road Salt Use**

Municipal. This category includes chlorides introduced by human and domestic wastes and various commercial usages. Values of from 5 g per day to 9 g per day in human wastes (4 lb per capita per yr to 7 lb per capita per yr) are reported (12). To allow for kitchen wastes as well as commercial and small manufacturing sources, a value of 20 lb per capita per yr is adopted. This is equivalent to a concentration in the order of 40 mg per l to 80 mg per l in average sewage depending on per capita water consumption.

Industrial. Information on the chlorides added to each basin by industries is incomplete. For two lakes yearly estimates have been made; for Lake Michigan an input of 1,800,000 lb per day (15) and for Lake Erie 13,700,000 lb per day (16). Both values are assumed to represent conditions in 1960.

With no historical data available, it is assumed that the industrial growth is proportional to the population for Lakes Michigan and Erie—the lakes...
settled earliest and the lakes with the most diverse industries. For Lakes Huron and Ontario, where chloride inputs are assumed to be mainly from chemical plants (4), the index of the value of chemical products, according to the U.S. Census of Manufacturers, is the basis for estimating industrial growth. A chronological plot of the index from 1950 to 1960 is shown in Fig. 6. The solid line represents the functional relationship used in this analysis.

Road Salt.—This source includes the chlorides in the salts used in the de-icing of streets and highways. It is assumed that the annual salt usage in a basin reaches the lake within the year it was added. Data on the amount of salt used by state highway departments and on toll roads are reported as total amounts per state (11). In this analysis the state highway department amounts are distributed according to the area of the state in the basin compared with that outside it. Toll road salt is proportioned on a road mileage basis.

The amounts per state are distributed according to the area of the state in the basin compared with that outside it. Toll road salt is proportioned on a road mileage basis.

Data on the amounts of salt used by cities and counties (2) are incomplete, e.g., of a total of 85 counties in one state, only 35 counties report values of salt use. The limited data available on city use is shown in Fig. 7, which is a plot of population versus salt usage. The solid lines shown in this figure are used as a basis for estimating usage by cities from which data is unavailable. A similar plot for county use indicated little correlation. Use by neighboring cities is the basis of assigning values to counties which did not report salt usage.

A tabulation of the various road salt inputs is found in Table 5 for the 1966-67 winter season. As a first approximation for the increase of the road salt input, it is assumed that salt was first used in 1930 and increased linearly to the 1960 value.

Inflow From Upstream Lakes.—This input accounts for the amount of man-made chlorides flowing into a lake from any upstream lake and thus applies to Lakes Huron, Erie and Ontario. For example, the input into Lake Huron from Lake Superior is the product of the concentration of Lake Superior and the mean flow of the St. Mary's River.

Other Sources.—Using the aforementioned inputs a total estimated quantity of chlorides brought into each lake may be calculated for 1960. A comparison of the input to each lake with the sum of the amount flowing out and the amount accumulated in the lake indicates that the inputs are less than the outputs in all cases. This difference may be due to other sources or sinks which are presently not known or to incomplete mixing and use of the total volume of each lake. It is probable that the major cause of the discrepancy is due to "other sources." The "other sources" listed in Table 6 are the amounts of chlorides which were needed in 1960 to provide a mass balance within a basin. Since they were probably composed of industrial or road salt inputs, they are assumed to be similar to the industrial development and population growth. The estimate of the percentage of chlorides known in 1960 is computed by dividing the input by the output. The low percentage of known chlorides in the Lake Huron basin is due to the lack of knowledge of the industrial inputs which derive primarily from salt mining in the Saginaw Valley (4). (From a recent analysis (17), it is estimated that a minimum of 1,800,000 lb per day of chlorides are introduced by industry in the Lake Huron basin).

**TABLE 5.—ROAD SALT IN GREAT LAKES BASIN**

<table>
<thead>
<tr>
<th>Lake</th>
<th>Toll roads</th>
<th>City</th>
<th>County</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reported</td>
<td>Estimated</td>
<td>Reported</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(8)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Superior</td>
<td>63</td>
<td>26</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>Michigan</td>
<td>212</td>
<td>28</td>
<td>52</td>
<td>97</td>
</tr>
<tr>
<td>Huron</td>
<td>137</td>
<td>16</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>Erie</td>
<td>231</td>
<td>211</td>
<td>109</td>
<td>96</td>
</tr>
<tr>
<td>Ontario</td>
<td>124</td>
<td>202</td>
<td>12</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>44</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>135</td>
<td>72</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>150</td>
<td>82</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>105</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>110</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>120</td>
<td>72</td>
</tr>
</tbody>
</table>

a 1,000 tons NaCl—Winter, 1966-1967.

**TABLE 6.—APPROXIMATE CHLORIDE BALANCE IN 1960**

<table>
<thead>
<tr>
<th>Lake</th>
<th>Estimated input</th>
<th>Estimated accumulation and outflow</th>
<th>Approximate other sources</th>
<th>Percentage of chloride sources known</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
</tr>
<tr>
<td>Superior</td>
<td>1.09</td>
<td>1.29</td>
<td>0.20</td>
<td>78</td>
</tr>
<tr>
<td>Michigan</td>
<td>4.29</td>
<td>5.50</td>
<td>1.51</td>
<td>57</td>
</tr>
<tr>
<td>Huron</td>
<td>4.40</td>
<td>5.58</td>
<td>1.11</td>
<td>53</td>
</tr>
<tr>
<td>Erie</td>
<td>2.63</td>
<td>7.93</td>
<td>0.60</td>
<td>53</td>
</tr>
<tr>
<td>Ontario</td>
<td>26.55</td>
<td>35.0</td>
<td>7.40</td>
<td>70</td>
</tr>
</tbody>
</table>

Values given in millions of pounds per day.

In general, the analysis proceeds from the upstream lakes to the downstream ones, and the responses of the upstream lakes are used as inputs to those downstream (see Eq. 4). The total waste discharged into a given lake is the sum of the local basin discharges and the wastes brought by inflow from the upstream lakes. Each basin is treated as having five component inputs: (1) Municipal; (2) Industrial; (3) Road de-icing salt; (4) "other sources"; and (5) inflow from upstream lakes. According to prior assumptions, the municipal, industrial, and "other" sources increase exponentially, while the increase of salt for road de-icing is linear. The functional forms of the input and the concentration response are listed in Table 1. The waste flowing into a downstream lake is the product of the concentration and outflow of the upstream lake. The functional form of this input, therefore, is determined by the upstream concentration and contains a constant term, a linear term and several exponential terms, the number of which depends on the lake location and the types of wastes discharged.

The total discharge of waste waters to any lake in the sum of components, W(t), as presented in Table 1. This sum, to include all possible forms, may be generally expressed as
\[ W_i = \left( \sum_{t=1}^{m_i} a_{i,j} e^{\omega t} \right) + b_i t + d_i \] .......................... (12)

in which \( i \) designates the individual components of the total waste input in a basin; \( m_i \) denotes the maximum number of exponential terms for a given component; and \( a_i, b_i, d_i \) and \( \omega \) are constants for a particular component. Eq. 1 may therefore be expressed as

\[ \frac{dc}{dt} = ac = \frac{1}{V} \sum_{t=1}^{n} \left( \sum_{t=1}^{m_i} a_{i,j} e^{\omega t} \right) + b_i t + d_i \] .......................... (13)

in which \( n \) is the number of waste components for a given lake. The solution of Eq. 13 is

\[ c = \frac{1}{V} \left[ P_0 - \sum_{t=1}^{n} \left( \sum_{t=1}^{m_i} a_{i,j} e^{\omega t} \right) + b_i t + d_i \right] \] .......................... (14)

in which

\[ A_{i,j} = \frac{a_{i,j}}{\alpha + \omega t, t} \] .......................... (15)

\[ B_i = \frac{b_i}{\alpha} \] .......................... (16)

and

\[ F = c_0 - \sum_{t=1}^{n} \left( \sum_{t=1}^{m_i} A_{i,j} \right) + D_i \] .......................... (17)

Eq. 14 is a representation of all the possible forms of solution \( c(t) \) to Eq. 13 as shown in Table 1. The quantity \( c_0 \) is the initial concentration due to development before the turn of the century. It is taken as the difference between the observed chloride concentration in 1900 and the natural background concentration.
centrations $c_1$. Thus, for the upstream lakes $c_o = 0$, whereas the Lakes Erie and Ontario $c_o = 4$ mg per l. It may be noted that two forms of the equation are required to express the chloride concentration in each lake: one from 1900 to 1930, the second from 1930 to 1960. The change is due to the introduction of road salt which is assumed to have been first used in 1930.

The influence of the components of the waste discharges on the concentration in Lake Michigan is shown in Fig. 8. It may be seen that the natural lake concentration is the largest component (3 mg per l) with the industrial contribution next. Municipalities cause very slight changes in the lake,

![Graph showing concentration changes in Lakes Michigan, Huron, Erie, and Ontario.](image)

FIG. 11.—COMPONENTS OF CHLORIDE CONCENTRATIONS

amounting to about 0.3 mg per l in 1960, and road salts amounted to about 0.75 mg per l. The latter is increasing at a relatively rapid rate. The residual concentration of about 0.75 mg per l is assigned to "other sources" rather than to incomplete mixing as previously indicated. By summing up the components, the total concentration of the lake is obtained. This total is shown as the solid line in the lower portion of Fig. 8, whereas the dashed line represents the sum of all components except the "other sources." Superimposed on this plot are observed data from the lake (4,9,15,20,32,23). These data were taken at various locations in the lake by several investigators. They represent single samples, spatial averages, or time averages, and some are both spatial and temporal averages. Even with this diversity a general trend is apparent.

Similar results for Lake Erie are presented in Fig. 9. In this case industrial sources within the local basin cause the greatest change in the lake concentration (over 12 mg per l since 1900). The effect of all upstream sources is shown as the Lake Huron inflow. Due to the small volume and large outflow of Lake Erie, the initial concentration of 4 mg per l is reduced to practically zero in ten years. As in the case of Lake Michigan, the municipal effect is very slight, about 0.5 mg per l in 1960.

A comparison between the calculated curve and the observed chloride concentrations appears in Fig. 10 for each lake and the various components of each curve are shown in Fig. 11. The natural background concentration is the principal component of the upstream lakes but is relatively insignificant for those downstream. Industry is the largest contributor to the chloride increase in the lakes. This condition is also characteristic of Lake Ontario where the principal component is the inflow from Lake Erie which is mainly composed of industrial chlorides. In all cases the effects of municipal chlorides are minimal.

Effects of Imposing Controls on Waste Load Inputs.—If controls are imposed on the chloride inputs to the Great Lakes in the future, the chloride concentration will be reduced. A useful reference point in making any comparison is the projected chloride concentration in a lake due to present discharges and growth rates. For Lake Erie, it may be seen in Fig. 12(a) that the projected chloride concentration is over 50 mg per l in the year 2000 with unrestricted growth of present sources. This concentration includes 25 mg
per l from industrial inputs in the local Lake Erie basin and 7 mg per l from industrial sources in the upstream lakes. It is assumed that the "other" sources within the Lake Huron basin are primarily industrial in nature. Assuming no chloride discharge of the local basin industry after 1970, the concentration in the year 2000 is reduced to 25 mg per l, as shown in Fig. 12. The local industry effect is reduced to substantially zero by 1980, illustrating the relatively rapid response of Lake Erie to changes within its own basin. The upstream industry still contributes 7 mg per l to the concentration in 2000. Assuming that the control is extended to industry in the upstream basins, the concentration of chloride in Lake Erie is further reduced to 20 mg per l in 2000 [Fig. 12(c)]. The upstream industrial effect changes from 7 mg per l to less than 2 mg per l in 2000.

FIG. 12.—EFFECT OF REMOVAL OF INDUSTRIAL CHLORIDES

Projections of the chloride concentration in Lakes Michigan, Erie and Ontario are shown in Fig. 13 for present growth rates and for the removal of industrial sources of chlorides. The projected curves in Fig. 13 are based on the assumption that "other" sources of Lakes Huron and Ontario are industrial. In all cases, the chloride concentration increases after an initial re-

FIG. 14.—DECREASE OF INDUSTRIAL CHLORIDE COMPONENTS DUE TO COMPLETE REMOVAL IN 1970

duction. Therefore, other man-made sources, such as the salt used for road de-icing, control the rate of increase of the chloride concentrations after industrial inputs are removed.

The removal of chloride sources from a drainage basin affects the concentration not only in the tributary lake but also in the downstream lakes. The reduction in chloride is inversely proportional to the detention time. Thus, it
is seen in Fig. 14 that the concentration of chlorides in Lake Michigan from industrial sources in its own drainage basin decreases slowly in Lake Michigan (upper figure) relative to the reduction of chlorides in Lake Huron from its industrial sources (lower portion). This effect is magnified in the downstream lakes. For example, the chlorides from Lake Michigan continue to increase the concentration in Lake Ontario to the year 2010, forty years after hypothetical control was imposed on the industrial discharges in the Lake Michigan drainage basin. On the other hand, the complete control of the industry in the Lake Huron basin in 1970 effects a relatively immediate response in the chlorides in Lake Ontario.

CONCLUSIONS

A simplified mathematical analysis of the concentration of chlorides in the Great Lakes has been presented. In spite of incomplete data on various sources, agreement between the calculated values and observations is reasonably good. The effect of sources in upstream drainage basins on the concentration in downstream lakes is significant. The characteristics of each lake determine its response to changes in discharges and control procedures. It is most important to quantize the magnitude of the various sources of potential pollutants, so that their relative importance may be assessed and control measures directed to the significant sources. A more detailed analysis of the sources is therefore required. When such data are available, it would be possible to analyze conditions in greater depth. Finally, this analysis may be used as a basis for estimating the effect of nonconservative constituents, such as nutrients, and provides a means for effecting a coordinated plan of quality control for each individual lake and the entire system.

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APPENDIX I.—REFERENCES

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APPENDIX II.—NOTATION

The following symbols are used in this paper:

\[ c = \text{concentration of waste in lake;} \]
GREAT LAKES

\[ c_0 = \text{waste concentration in lake in 1900 due to man's activities;} \]
\[ c_{r}, c_{p}, c_{l} = \text{waste concentration in runoff, precipitation and inflow;} \]
\[ c_l = \text{natural lake background concentration of waste;} \]
\[ I = \text{inflow to lake from upstream lake;} \]
\[ K = \text{first-order reaction coefficient for waste;} \]
\[ N = \text{population of lake basin;} \]
\[ N_0 = \text{population of lake basin in 1900;} \]
\[ P = \text{precipitation on lake;} \]
\[ Q = \text{flow out of lake basin;} \]
\[ R = \text{runoff within lake basin;} \]
\[ t = \text{time, in years;} \]
\[ t_0 = \frac{V}{Q} = \text{detention time of lake;} \]
\[ V = \text{volume of lake;} \]
\[ W = \text{mass rate of waste discharge; and} \]
\[ \alpha = K + 1/t_0 = \text{lake response parameter to waste.} \]

7470 WATER QUALITY MODEL OF CHLORIDES IN GREAT LAKES

KEY WORDS: chlorides; forecasting; Great Lakes; mathematical models; sanitary engineering; water pollution; water quality

ABSTRACT: The increase in the concentration of conservative substances in the Great Lakes is described by a simple time variable equation. The concentration of chlorides is related to the fresh water flow, the volumes of the lakes and the various sources—municipal, industrial, natural background and road de-icing. The increase in concentration since 1900 is presented and projections are made of anticipated concentrations based on various assumptions of control.