ORGANIC POLLUTION OF NEW YORK HARBOR—THEORETICAL CONSIDERATIONS

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Into the Upper Bay of New York Harbor flows the waters from the Hudson and East Rivers, Kill Van Kull, Gowanus Creek, and the wastewaters from the various treatment plants and outlets. The combined contribution of organic wastes from these sources produces a significant concentration of oxygen demanding wastes in the harbor waters and the associated reduction of dissolved oxygen, particularly during the summer months. It is the purpose of this paper to present the theoretical development defining the relationship between the BOD and DO in the Upper Bay and the Hudson River. Although this work is directed specifically toward conditions in the Harbor, it is necessary to analyze the effect of pollution received from the tributary rivers. Since the Hudson River is one of the most important factors in this regard, the body of water from The Narrows to the city line is considered, as shown in Figure 1. The pertinent sampling stations of the New York City Department of Public Works and the model stations of the U.S. Corps of Engineers are also shown in this figure. The present and future treatment plants are also indicated.

Theory

It has been observed that the concentration of BOD is approximately con-
stant throughout the Upper Bay. This condition is due to the mixing of tidal diffusion and to the number and location of wastewater outlets. Under a steady-state condition of freshwater discharge, organic loading and tidal turbulence, an equilibrium value of BOD is reached in a relatively short period of time. The equilibrium concentration may be determined from the equation, which defines the material balance of BOD in the harbor. The total weight of organic matter discharged daily from the harbor equals that which is added from the treatment plants and rivers minus that which is oxidized in the harbor. This relationship may be expressed as follows:

\[ L_B 2Q = W_1 + L_B Q - RV_B \]  

where

- \( L_B \) = BOD concentration (ultimate) in the harbor,
- \( 2Q \) = sum of the flows from the Hudson River, the waste treatment plants and outlets,
- \( W_1 \) = total pounds of BOD discharged to the harbor, not including that contributed by the Hudson River,
- \( L_B \) = Hudson River BOD,
- \( Q \) = Hudson River flow,
- \( R \) = Rate of oxygen utilization, and
- \( V_B \) = Volume of the Upper Bay.

If first-order kinetics are assumed for the BOD reaction, Equation 1 may be re-expressed:

\[ R = KL = \frac{W_1 + Q L_B - 2Q}{V_B} \]

or

\[ L = \frac{Q}{2Q + RV_B} \]

in which

- \( 2Q \) = total flow = \( Q + Q_B \),
- \( Q \) = Hudson River flow, and
- \( Q_B \) = discharges from the waste treatment plants.

In the development of Equations 2 and 3 consideration was not given to the diffusion between the Harbor and the various rivers discharging into it, since the concentrations in each of the bodies is of the same order of magnitude.

A condition of uniform concentration of BOD has also been observed in the Hudson River from the Battery to the city line. A material balance constructed in a similar fashion as in the Upper Bay, leads to the following equation:

\[ Q L_B = W_1 - RV_B \]
Assuming first-order kinetics and re-arranging:

\[ R = \frac{W_1 - QL}{V} = \frac{W_1}{V} - \frac{Q}{V} \frac{L}{L} \quad (5) \]

or

\[ L = \frac{W_1}{Q + KV} \quad (6) \]

in which

\( L \) = BOD concentration in the river,
\( W_1 \) = total pounds of BOD discharged to the river,
\( Q \) = Hudson River flow,
\( \dot{R} \) = rate of oxygen utilization,
\( V \) = volume of the river from the Battery to the city line, and
\( K \) = deoxygenation coefficient of the river waters.

The oxygen balance in the Upper Bay may be defined by taking into account the oxygen added by the tidal diffusion and respiration and the oxygen removed by deoxygenation due to the organic matter. Since the concentration of BOD is uniform throughout the bay, the longitudinal distribution of the oxygen uptake rate is a constant from the Narrows to the Battery. The differential equation which describes the combined action of these forces under the steady-state condition is:

\[ \dot{O} = \frac{E}{K_1} \frac{\partial c}{\partial x} + K_1 (c_s - c) - \dot{R} \quad (7) \]

Applying the boundary conditions that \( c = c_s \) at \( x = 0 \) and \( c = c_s \) at \( x = \infty \), Equation 7 integrates to:

\[ c = c_s - \frac{R}{K_1} \left[ 1 - e^{-\frac{K_1 x}{U}} \right] + (c_s - c) e^{-\frac{K_1 x}{U}} \quad (8) \]

in which

\( c \) = dissolved oxygen concentration,
\( c_s \) = saturation value of dissolved oxygen,
\( \dot{c} \) = concentration at \( x = 0 \),
\( x \) = longitudinal distance,
\( E \) = tidal diffusion coefficient, and
\( K_1 \) = resorption coefficient.

Equations 7 and 8 describe conditions in the main channel area of the harbor.

Littoral areas subject to localized pollution and less readily flushed by tidal action may have DO concentrations less than those in the main channel.

In the ease of the Hudson River, the velocity due to freshwater discharge is significant and must be included in the equations:

\[ \dot{O} = \frac{E}{K_1} \frac{\partial c}{\partial x} + \frac{\partial c}{\partial x} + K_1 (c_s - c) - \dot{R} \quad (9) \]

This equation may be integrated to:

\[ c = c_s - \frac{R}{K_1} (1 - e^\sqrt{\frac{K_1 x}{U \dot{R}}} \quad (10) \]

in which

\( j = \frac{U}{2E} \left[ 1 - \sqrt{1 + \frac{4 K_1 E}{U^2} \right] \quad (11) \]

\( U \) = velocity due to freshwater discharge of Hudson River.

Equations 8 and 10 are similar to those which were developed to define estuarine oxygen profiles for a single source of pollution (1).

The preceding equations define the steady-state distribution of DO and BOD. It is useful to consider also the non-steady distribution of a non-conservative substance. When a mass of such material is released instantaneously, the mass is dispersed by the tidal diffusion, translated by the advective effect of the freshwater discharge and reduced by a reaction characteristic of the particular substance. The non-steady state may be then defined by:

\[ \frac{\partial c}{\partial t} = \frac{E}{K_1} \frac{\partial c}{\partial x} - \dot{R} \quad (12) \]

The first, second, and third terms on the right-hand side of Equation 12 are respectively the diffusive, advective, and reactive effects. Constant coefficients and first-order kinetics are stipulated. The solution of this equation for an instantaneous release of a mass \( M \) which satisfies the boundary condition that \( c = 0 \) when \( t = 0 \) is:

\[ c = \frac{M}{2A \sqrt{\pi \dot{R}}} e^{-\frac{t}{2A \dot{R}}} \quad (13) \]

In Equation 13 \( A \) is the cross-sectional area of the channel and the other terms are as previously defined. If the velocity is negligible, as is assumed in the case of the Harbor, then Equation 13 becomes:

\[ c = \frac{M}{2A \sqrt{v E \dot{R}}} e^{-\frac{t}{2A \sqrt{v E \dot{R}}}} \quad (14) \]

Coefficients of Diffusion, Advection, and Reaction

**Diffusion**

The longitudinal distribution of the salinity in the Hudson River under the steady-state condition may be expressed by the following equation:

\[ \frac{t}{s_s} = e^{-\frac{s}{s_s} E} \quad (15) \]

![Graph](image-url)

**Graph**

- **Figure 2.** Relationship between flow and diffusion coefficient.

- **Legend**
  - CURVE A: Represents a different year 1954–1958
  - CURVE B: Indicates a different flow regime

- **Flow – 1000 CFS**

- **Log of 10 MILE SALINITY GRADIENT**

- **Log of 10 MILE FLOW**

- **Log of 10 MILE DIFFUSION COEFFICIENT**

- **Log of 10 MILE ADVECTIVE COEFFICIENT**

- **Log of 10 MILE REACTIVE COEFFICIENT**

- **Log of 10 MILE SOURCE LOCATION**
has not been taken into account in Equation 13. In employing this equation the concentrations of the lower salt water wedge were not included. A more complete analysis must take this factor into account. However, vertical stratification becomes less pronounced in the downstream direction and is practically eliminated in the Upper Harbor.

Advection

The advection coefficient is the velocity due to freshwater discharge and is calculated from the continuity equation. The mean cross-sectional area of the Hudson is taken as 135,000 sq ft. The discharge is that of the combined Hudson River flow at Mechanicville and Mohawk River flow at Cohoes, multiplied by a factor of 1.2 to allow for the additional drainage area from Mechanicville to the city line.

Calculation of Diffusion Coefficient

The diffusion coefficient for the Hudson River has been calculated in accordance with Equation 15 for various summer monthly flows from 1954 to 1958. The relationship between the coefficient and monthly discharge is shown in Figure 2 as curve A. The apparent high correlation between these variables is somewhat superficial since it is not related to the flow, was used to calculate the diffusion coefficient. A more fundamental measure of the correlation between flow and tidal diffusion is obtained by plotting the logarithm of the
10-mile chloridity ratio against flow as shown by curve B in Figure 2.

This procedure of calculating the diffusion coefficient could not be applied to the Upper Bay. Due to the large cross-sectional area of the bay, the velocity term is neither significant nor reliable. It was assumed that the order of magnitude of tidal diffusion is the same for both the river and the bay, and a value of 30 sq miles/day was assigned to the Upper Harbor.

A recent experimental study (3) of the dispersion patterns in a model of New York Harbor and Hudson River provided further data for the evaluation of diffusion coefficients. In these studies dye was released at various locations under different flow conditions. The concentration of dye at many stations and over many tidal cycles was measured at high-water slack. It was noted in the course of these experiments that dye was adsorbed on the surface of the model. Analysis of similar data on a model of the Delaware River Estuary indicated that this loss of dye could be described by a first-order reaction. A reaction coefficient of 0.12/day for the dye adsorption phenomenon was assigned for the Hudson River and Upper Harbor. Values of the diffusion coefficient were taken from Figure 2 for the particular flow under which the experimental work was conducted. The value of 135,000 sq ft was employed for the cross-sectional area. Equation 3 was employed to calculate the temporal distribution of the dye at a station 1 mile downstream from the Battery (Station 0). The observed data and calculated profiles are shown in Figure 3 for Hudson River flows of 6,000 and 12,000 cfs. Figure 4 presents the curves calculated from Equation 14 and observed data at stations in the harbor. The diffusion coefficient was taken as 30 sq miles/day, as previously described, and the area as 300,000 sq ft.

The freshwater discharge in these experiments was 6,000 cfs. The figures show the temporal distributions at two stations in the city; one located approximately 21 miles upstream from the Narrows (Station 4). The location of these stations is shown in Figure 1. The location of the release point of the dye of these experiments was high-water slack. All dye measurements were made at high-water slack. Therefore, in order to employ the mathematical model developed above, it was necessary to transfer the release points to an equivalent high-water slack. This position was located by taking the average flood current and multiplying by the tidal cycle of approximately 6.2 hr. The release point was transposed approximately 7 miles for the Harbor release experiments and approximately 9 miles for the Hudson River release.

Reaeration Coefficient

The reaeration coefficient may be approximated by the following formula:

$$K_r = \frac{D_U}{H} \quad (16)$$

in which

- $D_U$ = diffusion coefficient of oxygen in sea water
- $U_*$ = average tidal velocity
- $H$ = average depth

Based on a 20°C temperature and the values of depth and tidal velocity for the Bay and the Harbor tabulated below the reaeration coefficients were calculated. These values are shown in Table 1 with the assigned values of $K$ at 20°C. After inspection of the meteorological data on summer wind velocities and consideration of the relatively large exposed water surface of the harbor, the reaeration coefficient was increased by 15 percent to 0.23 per day. This increase is comparable to that reported for the Thames Estuary (4). The coefficient of deamination $K$ for the Bay is assigned as 0.23 per day at 20°C, which is within the normal range for polluted seawater. In view of the fact that the waste in the Hudson is fresher and the salt-water concentration less than these corresponding characteristics in the Bay, the coefficient is increased to 0.25 per day at 20°C for the Hudson River. It is appreciated that no sharp line of demarcation exists between these two bodies. This condition is assumed for simplicity of analysis and ease of computation. Furthermore, it is assumed that the 5-day BOD reported is 75 percent of the ultimate. The periods for which analyses were made were based on a study of the temperature data. When the temperature changed more than 2°C per day, the running mean was determined by a study of the temperature data. When the temperature changed more than 2°C per day, the running mean was determined by a study of the temperature data. The average values of DO and BOD were computed for each period. It is for

**TABLE I. Estuary Coefficients**

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<thead>
<tr>
<th>Year</th>
<th>River</th>
<th>Temp</th>
<th>5-Day BOD</th>
<th>Sec. Water</th>
<th>Satur. Time</th>
<th>Temp</th>
<th>5-Day BOD</th>
<th>Sec. Water</th>
<th>Satur. Time</th>
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<td>79</td>
<td>8.0</td>
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</tbody>
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*Percent seawater for Hudson equals log mean value of top sample from Battery to City Line; all other values are arithmetic means for period. Periods 1 to 4 are approximately equivalent to the months of June to Sept., respectively. Data from N.Y.C. Department of Public Works except for a period in 1958 identified by I.S.C. (Interstate Sanitation Commission).
tuous that these intervals approximately coincided with the monthly periods of June, July, Aug., and Sept. An example of these data is shown in Figure 5. The Hudson River flow at Mechanicville and the Mohawk River flow at Cohoes were added and the sum multiplied by 1.2 to allow for drainage area below these stations. The time of travel of 30 days is assumed between Mechanicville and the city line. It is observed that three values of the BOD were not included in the averages indicated in Figure 5. Periodically values of BOD and DO much higher than the average were reported and these were not included in the average. Approximately 10 percent of the BOD data and less than 5 percent of the DO data were eliminated on this basis. It is significant to note that these DO values were usually above saturation and at the upstream stations. In calculating the average BOD for the Harbor the data collected in the vicinity of the Passaic Valley Sewage Treatment Plant outfall (Station 6) were not included. Both top and bottom samples were included in the averages for the Harbor but only the top samples were used for the Hudson, as described previously.

A summary of the important data and the assigned time periods are shown in Table II.

**Analysis of Data**

The New York City Department of Public Works has been sampling the harbor and river waters during the summer months for a number of years. Sampling has been performed on approximately a weekly basis. These data were made available by the Department for this analysis. An attempt was first made to analyze the data on the basis of the slack periods as described by Kaplovsky (5) in a report on the Delaware Estuary. In an analysis of the cyclical variation of coliform bacteria (6) it was shown that the maximum and minimum MPN at a station in New York Harbor occurred at slack periods. However, there is some doubt that slack sampling technique could properly be applied in case of the Hudson River.

In a recent report on the Hudson River (7) the data of the New York City Department of Public Works were summarized and averaged. Examples of these data for various stations are shown in Figure 6. These data are averages of all samples collected at the same time of the tidal cycle regardless of flow and temperature. The correlation between the time in the tidal cycle and the dissolved oxygen concentration was significant. The variation at Station 5 is the least, because this is the location of the minimum point of the dissolved oxygen profile. However, the slack periods do not coincide with the maximum and minimum values of the dissolved oxygen. It was indicated in this report (7) that this may be due to the influence of the Harlem River, whose tidal cycle does not coincide with that of the Hudson. The minimal variation of the BOD indicates a temporal as well as spatial uniformity of the concentration of oxygen in the water material in the Hudson River. This condition is primarily due to the relatively uniform discharge of wastewater from the western sections of Manhattan and the Bronx. Figure 1 shows that the treatment plant for this area is presently under design and at the present time the wastewaters from combined sewers are discharged directly to the River from the Battery to the city line. The tidal motion and mixing also account for the spatial uniformity of BOD. The data shown in Figure 6 are surface samples. The bottom samples at Station 5 followed the same pattern, but at Station 1 the effects of the stratification were more marked. The data shown in this figure are composites of all the data collected at the particular stations from 1956 to 1959.
In view of the nature of available data, it was assumed that average values over approximately a monthly period would provide a reasonable mean of BOD and DO concentrations.

The BOD values were then calculated in accordance with Equations 3 and 6 and the DO profiles in accordance with Equations 8 and 10 for the Upper Bay and the Hudson River, respectively.

The comparisons between the calculated and observed BOD are shown in Figures 7 and 8.

The values for the calculated BOD in Figure 7 were determined by the equation:

\[ \text{BOD}_{\text{calculated}} = \frac{W + Q_f L}{Q_B + Q_1 + \frac{W}{4K}} \]  

\[ \text{eq}(5) \]

\[ \text{eq}(3) \]

\[ K_B = 0.26/\text{DAY} \]

\[ K_D = 0.30/\text{DAY} \]

\[ \text{eq}(8) \]

\[ \text{eq}(10) \]

\[ K_B = 0.26/\text{DAY} \]

\[ T = 23^\circ \text{C} \]

\[ K_D = 0.11/\text{DAY} \]

\[ T = 24^\circ \text{C} \]

\[ \text{DO} - \text{MG/L} \]

\[ \text{NYCDPW STATIONS} \]

\[ \text{MILES FROM NARROWS} \]

\[ \text{OBSERVED DATA} \]

\[ \bullet \text{ 1st PERIOD} \]

\[ \square \text{ 2nd PERIOD} \]

\[ \triangle \text{ 3rd PERIOD} \]

\[ \diamond \text{ 4th PERIOD} \]

\[ \text{BOTTOM} \]

\[ \text{AVG VALUE} \]

\[ \text{TOP} \]

\[ \text{CITY LINE} \]

\[ \text{FIGURE 10.--Dissolved oxygen profiles, 1954 and 1956.} \]
where

\[ W = \text{lb/day BOD (excluding the Hudson)} \]
\[ Q_B = \text{Hudson River flow (average monthly flow)} \]
\[ L_B = \text{Hudson River BOD (at the Battery)} \]
\[ Q_T = \text{flow from treatment plants} = 650 \text{ mgd} \]
\[ V_B = \text{volume of Upper Bay} = 12,100 \text{ mcft} \]
\[ K = \text{oxidation rate} = 0.23/\text{day at } 20^\circ C \]

The values of the calculated BOD in Figure 8 are determined by the equation:

\[ \text{BOD}_{\text{calculated}} = \frac{W}{Q_B + V_B K} \]

where

\[ W = \text{lb/day BOD (525,000 lb/day)} \]
\[ Q_B = \text{Hudson River flow} \]
\[ V_B = \text{volume of Hudson (city line to Battery)} = 13,400 \text{ mcft} \]
\[ K = \text{oxidation rate} = 0.23/\text{day at } 20^\circ C \]

Each point represents the average value of the summer months for the years 1954 to 1958 inclusive. It was assumed that one-half of the pollution load of the proposed Red Hook and Newtown Creek plants is discharged to the Upper Harbor, and the remaining half is ultimately flushed to Long Island Sound. The general pattern of the BOD concentration computed is substantially borne out by the measurements. Although the agreement is not perfect, the deviations are within the usual variations for BOD values of this magnitude. The following procedure was used in calculating the DO profiles.

1. The saturation value of oxygen in the Hudson was determined for the given temperature and mean chloride concentration from the Battery to the city line.
2. The initial DO concentration was taken from the observed data.
3. The desaturation and reoxygenation coefficients were corrected to the appropriate temperature.
4. For the given Hudson River runoff, the diffusion coefficient and the velocity were calculated.
5. The observed 5-day BOD average was converted to the ultimate value \( L_B \) and the DO values are calculated in accordance with Equation 10.
6. Saturation value in the Bay was usually the same as for the Hudson River because of the countering effect of the reduced temperature and increased chloride concentration.
7. A similar procedure was followed in computing the DO profile in the Upper Harbor in accordance with Equation 8.

Figure 9 shows the calculated profiles of chloride, BOD, and DO for the month of Aug., 1957. The chloride profile was calculated from Equation 15 and the BOD concentration from Equations 3 and 6. The DO profile was determined as described above. Additional examples of DO profiles are shown in Figure 10 for 1954 and 1956. In 1958 the Interstate Sanitation Commission conducted surveys of the Harbor and these data with the calculated curves are shown in Figure 11 (8).

**Conclusions**

1. The average values, covering a period of about one month, appear to present a realistic pattern of pollution of the Upper Bay and Hudson River. It may be concluded that variations in dissolved oxygen concentration about the monthly mean are in the order of 0.5 to 1.0 mg/l.
2. The fluctuations of dissolved oxygen about the mean are primarily associated with the cyclical variation of the tidal motion and secondarily to variations in runoff waste discharges and temperature.
3. In spite of the simplifying assumptions, the general pattern of BOD concentration is substantially borne out by the calculations. In this regard, the magnitude of the contributing
population, the pollution load, the coefficient of desaturation and the conversion factor for the 3-day to ultimate BOD should be confirmed by a more complete study.

4. The general pattern of the dissolved oxygen profiles as theoretically determined agrees reasonably well with observed monthly mean values. Only the dissolved oxygen concentration in the upper zone of the Hudson River is considered. The bottom values are less due to diffusion from the downstream area and vertical stratification. This latter factor was not taken into account in the theoretical development. In addition, interdiffusion between the Hudson and East Rivers and the Upper Bay was not considered. These factors and the possible photosynthetic contribution at the upstream stations should be taken into account in future work. In spite of these shortcomings, it may be concluded that the proposed equations provide a reasonable and qualitative basis for the definition of present conditions and for extrapolation to future conditions.

5. From the preliminary analysis of the data, the dye studies in the model of New York Harbor and the Hudson River provide appropriate data for the determination of the diffusional patterns. Further analysis is being conducted in this regard.

Acknowledgments

The theoretical work presented in this paper was supported in part by a research grant from the National Science Foundation. Analyses of data on dissolved oxygen profiles and BOD were performed in connection with a study of New York Harbor by Lockwood, Kessler, and Bartlett, Consultants for the Water Pollution Control Board of New York State. The cooperation of William A. O’Leary and Robert Shapiro, respectively, Director and Chief of Laboratories, Division of Sewage Disposal, New York City Department of Public Works, is gratefully acknowledged.

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