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STRENGTH DESIGN OF CONCRETE MASONRY

TEK 14-4A
Structural (2002)

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INTRODUCTION

Loadbearing concrete masonry walls can be designed using one of several methods in accordance with *Building Code Requirements for Masonry Structures* (ref. 1): empirical design, strength design or allowable stress design. This TEK provides a basic overview of design criteria and requirements for concrete masonry structures designed using the strength design provisions contained in Chapter 3 of *Building Code Requirements for Masonry Structures* (ref. 1). For empirical and allowable stress design provisions, the reader is referred to TEK 14-8A *Empirical Design of Concrete Masonry Walls* (ref. 2) and TEK 14-7A *Allowable Stress Design of Concrete Masonry* (ref. 3), respectively. Tables, charts and additional design aids specific to the design of various concrete masonry elements can be found in other related TEK.

Strength design is based on the following design assumptions in conjunction with basic principles of engineering mechanics (ref. 1):

- Plane sections before bending remain plane after bending. Therefore, strain in the masonry and in reinforcement, if present, is directly proportional to the distance from the neutral axis.
- For unreinforced masonry, the flexural stresses in the masonry are assumed to be directly proportional to strain. For reinforced masonry, the tensile strength of the masonry is neglected when calculating flexural strength, but considered when calculating deflection.
- The units, mortar, grout and reinforcement for reinforced masonry act compositely to resist applied loads.
- The nominal strength of masonry cross-sections for combined flexure and axial load is based on applicable conditions of equilibrium.
- The maximum masonry compressive stress is $0.80f'_m$ for both reinforced and unreinforced masonry.
- For reinforced masonry, the maximum usable strain, ε_{mu}, at the extreme compression fiber of concrete masonry is 0.0025.
- For reinforced masonry, reinforcement stresses below the specified yield strength, f_y, are taken equal to the modulus of elasticity of the reinforcement, E_s, times the steel strain ε_s. For

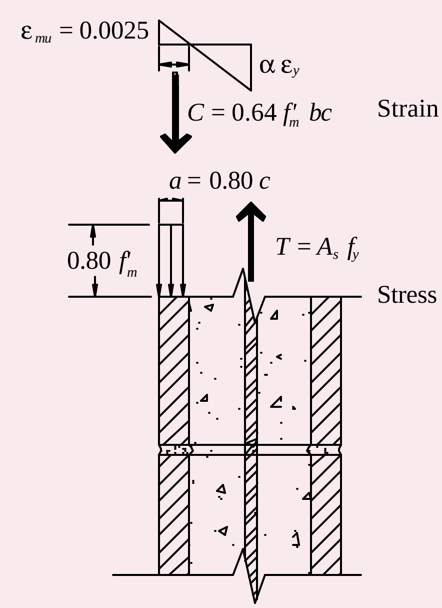

Figure 1—Stress and Strain Distribution for Reinforced Masonry

strains greater than that corresponding to f_y, stress in the reinforcement is taken equal to f_y.

- For reinforced masonry, the compressive stress is rectangular and uniformly distributed over an equivalent compression zone, bounded by the compression face of the masonry with a depth of $a = 0.80c$.

Based on the assumed design model outlined above, the internal distribution of stresses and strains is illustrated in Figure 1 for a reinforced masonry element.

DESIGN CRITERIA

Using strength design, the design strength of a masonry element is compared to the required strength (indicated by the subscript u), which includes load factors to account for the uncertainty in predicting design loads. The required strength is based on the strength design load combinations of the applicable building code. When the building code does not contain such load combinations, masonry structures are designed to resist the combination of loads specified in *Minimum Design Loads for Buildings and Other Structures* (ref. 4).

The design strength of masonry is the nominal strength (indicated by the subscript n) multiplied by an appropriate strength reduction factor ϕ. The design is acceptable when the design strength equals or exceeds the required strength (i.e., when $\phi M_n \geq M_u$).

Table 1—Modulus of Rupture, f_r, psi (kPa) (ref. 1)

Direction of flexural tensile stress & masonry type	Mortar types: Portland cement/lime or mortar cement, M or S	Portland cement/lime or mortar cement, N	Masonry cement or air entrained portland cement/lime, M or S	Masonry cement or air entrained portland cement/lime, N
Normal to bed joints (walls spanning vertically) in running or stack bond:				
Solid units	100 (689)	75(517)	60 (413)	38 (262)
Hollow units[a]				
Ungrouted	63 (431)	48 (331)	38 (262)	23 (158)
Fully grouted	170 (1,172)	145 (999)	103 (710)	73 (503)
Parallel to bed joints (walls spanning horizontally) in running bond :				
Solid units	200 (1,379)	150 (1,033)	120 (827)	75 (517)
Hollow units				
Ungrouted & partially grouted	125 (862)	95 (655)	75 (517)	48 (331)
Fully grouted	200 (1,379)	150 (1,033)	120 (827)	75 (517)
Parallel to bed joints in stack bond:	0 (0)	0 (0)	0 (0)	0 (0)

[a] For partially grouted masonry, f_r is determined by linear interpolation between fully grouted hollow units and ungrouted hollow units based on the amount of grouting.

Strength Reduction Factors

To account for uncertainties in construction, material properties, calculated versus actual strength and anticipated failure modes, the nominal strength of a masonry element is multiplied by an appropriate strength reduction factor, ϕ. The strength reduction factors are used in conjunction with the load factors applied to the design loads. The values of the strength reduction factors for various types of loading conditions are as follows:

- For reinforced masonry elements subjected to flexure or axial loads; $\phi = 0.90$.
- For unreinforced masonry elements subjected to flexure or axial loads; $\phi = 0.60$.
- For masonry elements subjected to shear loads; $\phi = 0.80$.
- For bearing on masonry elements; $\phi = 0.60$.

UNREINFORCED MASONRY

For unreinforced masonry, the masonry assembly (units, mortar and grout, if used) is designed to carry all applied stresses. The additional capacity from the inclusion of reinforcing steel, such as reinforcement added for the control of shrinkage cracking or prescriptively required by the code, is neglected. Because the masonry resists both tension and compression stresses resulting from applied loads, the masonry must be designed to remain uncracked.

Unreinforced Nominal Flexural Strength

The nominal flexural tensile strength of unreinforced concrete masonry is given by the modulus of rupture as prescribed in *Building Code Requirements for Masonry Structures* (ref. 1), which varies with the direction of span, mortar type, bond pattern and percentage of grouting as shown in Table 1. For walls spanning horizontally between supports, the code conservatively assumes that stack bond masonry has no flexural bond strength across the mortared head joints, thus only the grout area (for horizontally grouted cores) is used.

For masonry elements subjected to a factored bending moment, M_u, and a compressive axial force, P_u, the resulting flexural bending stress is determined using Equation 1.

$$F_u = \frac{M_u t}{2 I_n} - \frac{P_u}{A_n} \qquad \text{Eqn. 1}$$

TEK 14-1 *Section Properties of Concrete Masonry Walls* (ref. 5) provides typical values for net moment of inertia, I_n, and cross-sectional area, A_n, for various wall sections. If the value of the bending stress, F_u, given by Equation 1 is positive, then the masonry section is controlled by tension and the modulus of rupture values of Table 1, reduced by the appropriate strength reduction factor, must be satisfied. Conversely, if F_u as given by Equation 1 is negative, the masonry section is in compression and the design compressive stress of $0.80f'_m$ applies. When using axial load to offset flexural bending stresses, only dead load or other permanent loads should be included in P_u.

Unreinforced Nominal Axial Strength

When unreinforced masonry walls are subjected to compressive axial loads only, the nominal axial strength, P_n, is determined using equation 2 or 3, as appropriate.

For elements with h/r not greater than 99:

$$P_n = 0.80\left[0.80\, A_n f'_m \left(1 - \left(\frac{h}{140r}\right)^2\right)\right] \qquad \text{Eqn. 2}$$

For elements with h/r greater than 99:

$$P_n = 0.80\left[0.80\, A_n f'_m \left(\frac{70r}{h}\right)^2\right] \qquad \text{Eqn. 3}$$

Unreinforced Nominal Shear Strength

Shear stresses on unreinforced masonry elements are calculated using the net cross-sectional properties of the masonry in the direction of the applied shear force using the following relation:

$$F_{vu} = \frac{V_u Q_n}{I_n b} \qquad \text{Eqn. 4}$$

Equation 4 is applicable to the determination of both in-plane and out-of-plane shear stresses. Because unreinforced masonry is designed to remain uncracked, it is not necessary to perform a cracked section analysis to determine the net cross-sectional area of the masonry. The nominal shear strength, V_n, of an unreinforced masonry section is the least of:

1. $3.8A_n\sqrt{f'_m}$ psi ($0.375\,A_n\sqrt{f'_m}$ MPa)
2. $300A_n$ psi ($0.83A_n$ MPa)
3. a. For running bond not solidly grouted and for stack bond masonry with open end units and grouted solid,
   $56A_n + 0.45N_v$ ($0.26A_n + 0.3N_v$ MPa)
   b. For solidly grouted running bond masonry,
   $90A_n + 0.45N_v$ ($0.414A_n + 0.3N_v$ MPa)
   c. For stack bond masonry with other than open end units grouted solid,
   $23A_n$ ($0.103A_n$ MPa)

REINFORCED MASONRY

The design of reinforced masonry in accordance with *Building Code Requirements for Masonry Structures* (ref. 1) neglects the tensile resistance provided by the masonry units, mortar and grout in determining the strength of the masonry assemblage. Thus, for design purposes, the portion of masonry subject to net tensile stress is assumed to have cracked, transferring all tensile forces to the reinforcement.

Strength design of reinforced masonry is based on the specified yield strength of reinforcement, f_y, which is limited to 60,000 psi (413.7 MPa). The actual yield strength of the reinforcement is limited to 1.3 times the specified yield strength. The compressive resistance of steel reinforcement is not permitted to be used unless lateral reinforcement is provided in compliance with Chapter 2 of *Building Code Requirements for Masonry Structures* (ref. 1).

Using strength design, reinforcing bars used in masonry shall not be larger than No. 9 (M #29). Further, the nominal bar diameter is not permitted to exceed one-eight of the nominal member thickness or one-quarter of the least clear dimension of the cell, course or collar joint in which it is placed. The area of reinforcing bars placed in a cell or in a course of hollow unit construction shall not exceed four percent of the cell area.

Maximum Reinforcement Percentage

To provide for a prescribed level of ductility in the event of failure, the amount of reinforcement permitted in reinforced masonry construction is limited. The maximum reinforcement ratio, ρ_{max}, is limited in accordance with Equation 5 or 6, as appropriate. Equation 5 applies to masonry cross sections that are fully grouted or where the neutral axis falls within the face shell of the masonry units in partially grouted construction. When the neutral axis falls within the cores of partially grouted construction, Equation 6 should be used.

$$\rho_{max} = \frac{0.64\, f'_m\left(\frac{e_{mu}}{e_{mu} + \alpha e_y}\right) - \frac{P}{bd}}{1.25\, f_y} \qquad \text{Eqn. 5}$$

$$\rho_{max} = \frac{0.64\, f'_m\left(\frac{e_{mu}}{e_{mu} + \alpha e_y}\right)\left(\frac{b_w}{b}\right) + 0.80\, f'_m t_{fs}\left(\frac{b - b_w}{bd}\right) - \frac{P_u}{bd}}{1.25\, f_y}$$

Eqn. 6

Where the tension reinforcement yield strain factor, α, shall be as follows:

(a) α = 1.3 for walls subjected to out-of-plane forces and designed using an *R* value greater than 1.5.

(b) α = 5.0 for walls subjected to in-plane forces, for columns and for beams designed using an *R* value greater than 1.5.

(c) α = 2.0 for masonry structures designed using an *R* value equal to or less than 1.5.

Reinforced Nominal Axial Strength

The nominal axial strength, P_u, of masonry walls, piers and columns, modified to account for the effects of slenderness, is determined using equation 7 or 8, as appropriate.

For elements with h/r not greater than 99:

$$P_n = 0.80\left[0.80\ f'_m\left(A_n - A_s\right) + f_y\left(A_s\right)\right]\left(1 - \left(\frac{h}{140r}\right)^2\right) \quad \text{Eqn. 7}$$

For elements with h/r greater than 99:

$$P_n = 0.80\left[0.80\ f'_m\left(A_n - A_s\right) + f_y\ A_s\right]\left(\frac{70r}{h}\right)^2 \quad \text{Eqn. 8}$$

The area of reinforcing steel, A_s, is to be included in the calculation of nominal axial strength only if it is laterally confined in accordance with the provisions of Chapter 2 of *Building Code Requirements for Masonry Structures* (ref. 1).

Reinforced Nominal Flexural Strength

The nominal flexural strength, M_n, of a masonry element is determined in accordance with the following requirements. In addition, the nominal flexural strength at any section along a member shall not be less than one-fourth of the maximum nominal flexural strength at the critical section.

When axial loads are not present, or are conservatively neglected as may be appropriate in some cases, there are several circumstances to consider in determining the nominal flexural strength of reinforced masonry walls. For a fully grouted element, the internal moment arm between the resulting compressive and tensile forces is resolved to determine the resisting capacity of the section. Partially grouted walls are analyzed in the same way, but with the additional consideration of the ungrouted cores. For partially grouted masonry there are two types of behavior to consider.

1. The first case applies when the neutral axis (the location of zero stress) lies within the compression face shell. In this case, the wall is analyzed and designed using the procedures for a fully grouted wall.
2. The second type of analysis occurs when the neutral axis lies within the core area, rather than the compression face shell. For this case, the portion of the ungrouted cells must be deducted from the area of masonry capable of carrying compression stresses.

The location of the neutral axis depends on the spacing of the reinforcing steel as well as the reinforcement ratio, r, and the distance between the reinforcement and the extreme compression fiber, d.

When analyzing partially grouted walls, it is typically assumed that the neutral axis lies within the compression face shell, as the analysis is more straightforward. Based on this assumption, the resulting value of c is calculated. If it is determined that the neutral axis lies outside the compression face shell, the more rigorous tee beam analysis is performed. Otherwise, the rectangular beam analysis is carried out.

Rectangular Beam Analysis

For fully grouted masonry elements and for partially grouted masonry walls with the neutral axis in the compression face shell, the nominal flexural strength, M_n, is calculated using equations 9 and 10 as follows:

$$M_n = \left(A_s f_y + P_u\right)\left(d - \frac{a}{2}\right) \quad \text{Eqn. 9}$$

Where

$$a = \frac{P_u + A_s f_y}{0.80\ f'_m b} \quad \text{Eqn. 10}$$

Tee Beam Analysis

For partially grouted masonry walls where the neutral axis is located within the cores, the nominal flexural strength, M_n, is calculated using equations 11, 12, and 13 as follows:

$$M_n = \left(A_s f_y + P_u\right)\left(d - X\right) \quad \text{Eqn. 11}$$

$$X = \frac{\dfrac{b\left(t_{fs}^{\ 2}\right)}{2} + b_w\left(a - t_{fs}\right)\left(t_{fs} + \dfrac{a - t_{fs}}{2}\right)}{bt_{fs} + b_w\left(a - t_{fs}\right)} \quad \text{Eqn. 12}$$

$$a = \frac{P_u + A_s f_y}{0.80\ f'_m b_w} - t_{fs}\left(\frac{b}{b_w} - 1\right) \quad \text{Eqn. 13}$$

To account for deflection resulting from the application of out-of-plane loads and the additional bending moment due to eccentrically applied axial loads, the factored bending moment at the mid-height of a simply supported wall under uniform loading is required to be determined by Equation 14.

$$M_u = \frac{w_u h^2}{8} + P_{uf}\frac{e_u}{2} + P_u d_u \quad \text{Eqn. 14}$$

Where

$$P_u = P_{uw} + P_{uf} \quad \text{Eqn. 15}$$

Reinforced Nominal Shear Strength

Shear acting on reinforced masonry members is resisted by the masonry and shear reinforcement, if provided, in accordance with the following:

$V_n = V_m + V_s$ Eqn. 16

Where V_n is not permitted to exceed the values given by Equations 17 or 18, as appropriate.

Where M/Vd_v is less than or equal to 0.25:

$$V_n \le 6A_n\sqrt{f'_m} \quad \text{Eqn. 17}$$

Where M/Vd_v is greater than 1.00:

$$V_n \le 4A_n\sqrt{f'_m} \quad \text{Eqn. 18}$$

For values of M/Vd_v between 0.25 and 1.00, the maximum value of V_n may be interpolated.

The nominal shear strength provided by the masonry is determined in accordance with the following.

$$V_m = \left[4.0 - 1.75\left(\frac{M}{Vd_v}\right)\right]A_n\sqrt{f'_m} + 0.25\,P \quad \text{Eqn. 19}$$

$$\left(metric: V_m = 0.83\left[4.0 - 1.75\left(\frac{M}{Vd_v}\right)\right]A_n\sqrt{f'_m} + 0.25\,P\right)$$

Where the value of M/Vd need not be taken greater than 1.0.

When shear reinforcement is incorporated into reinforced masonry construction, the shear strength provided by the reinforcement is calculated in accordance with the following.

$V_s = 0.5(A_v/s)f_y d_v$ Eqn. 20

NOTATIONS

A_n = net cross-sectional area of masonry, in.^{2} (mm^2)
A_s = effective cross-sectional area of reinforcement, in.^{2} (mm^2)
A_v = cross-sectional area of shear reinforcement, in.^{2} (mm^2)
a = depth of an equivalent compression zone at nominal strength, in. (mm)
b = width of section, in. (mm)
b_w = for partially grouted walls, width of grouted cell plus each web thickness within the compression zone, in. (mm)
C = resultant compressive force, lb (N)
c = distance from the fiber of maximum compressive strain to the neutral axis, in. (mm)
d = distance from the extreme compression fiber to centroid of tension reinforcement, in. (mm)
d_v = actual depth of masonry in direction of shear considered in. (mm)
E_s = modulus of elasticity of reinforcement, psi (MPa)
e_u = eccentricity of P_{uf} in. (mm)
F_u = net flexural bending stress, psi (MPa)
F_{vu} = shear stress on unreinforced masonry elements
f'_m = specified compressive strength of masonry, psi (MPa)
f_r = modulus of rupture, psi (MPa)
f_y = specified yield strength of reinforcement, psi (MPa)
h = effective height of masonry element, in. (mm)
I_n = moment of inertia of net cross-sectional area of a member, in.^{4} (mm^4)
M = unfactored maximum calculated bending moment at the section under consideration, in.-lb (N-mm)
M_n = nominal moment strength, in.-lb (N-mm)
M_u = factored moment, in.-lb (N-mm)
N_v = compressive force acting normal to the shear surface, lb (N)
P = unfactored axial load, lb (N)
P_n = nominal axial strength, lb (N)
P_u = factored axial load, lb (N)
P_{uf} = factored load from tributary floor or roof areas, lb (N)
P_{uw} = factored weight of wall area tributary to wall section under consideration, lb (N)
Q_n = first moment about the neutral axis of a section of that portion of the net cross section lying between the neutral axis and extreme fiber, in.^{3} (mm^3)
R = seismic response modification factor, determined using ASCE 7 (ref. 4)
r = radius of gyration, in. (mm)
s = spacing of shear reinforcement, in. (mm)
T = tension in reinforcement, lb (N)
t = specified thickness of masonry element, in. (mm)
t_{fs} = concrete masonry face shell thickness, in. (mm)
V = unfactored shear force, lb (N)
V_m = shear strength provided by masonry, lb (N)
V_n = nominal shear strength, lb (N)
V_s = shear strength provided by shear reinforcement, lb (N)
V_u = factored shear, lb (N)
w = out-of-plane factored uniformly distributed load, lb/in. (N/mm)
X = for partially grouted masonry, the distance from extreme compression fiber to the centroid of the compression resultant, in. (mm)
α = tension reinforcement yield strain factor
δ_u = deflection due to factored loads, in. (mm)
ε_{mu} = maximum usable compressive strain of masonry
e_s = strain in the reinforcement
ε_y = yield strain of reinforcement
ρ_{max} = maximum reinforcement ratio
ϕ = strength reduction factor

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