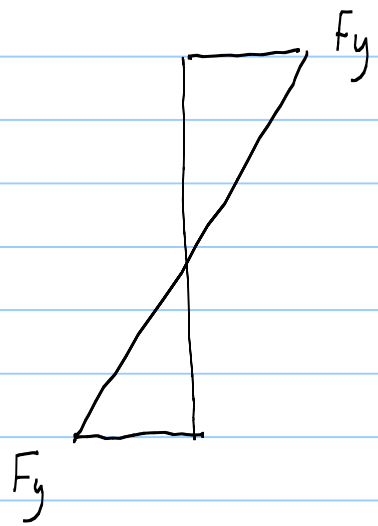
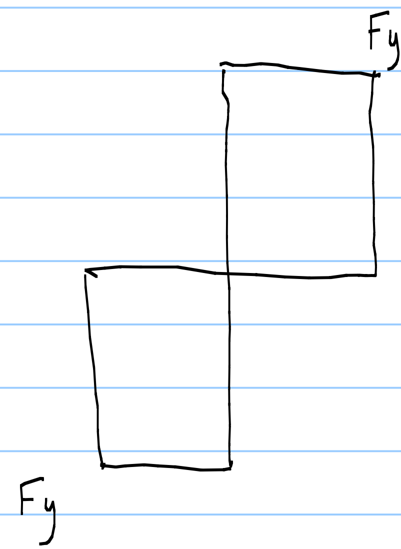
Note Title 4/26/2007

Announcement - No Office Hours Today





$M = M_y$ = yield moment

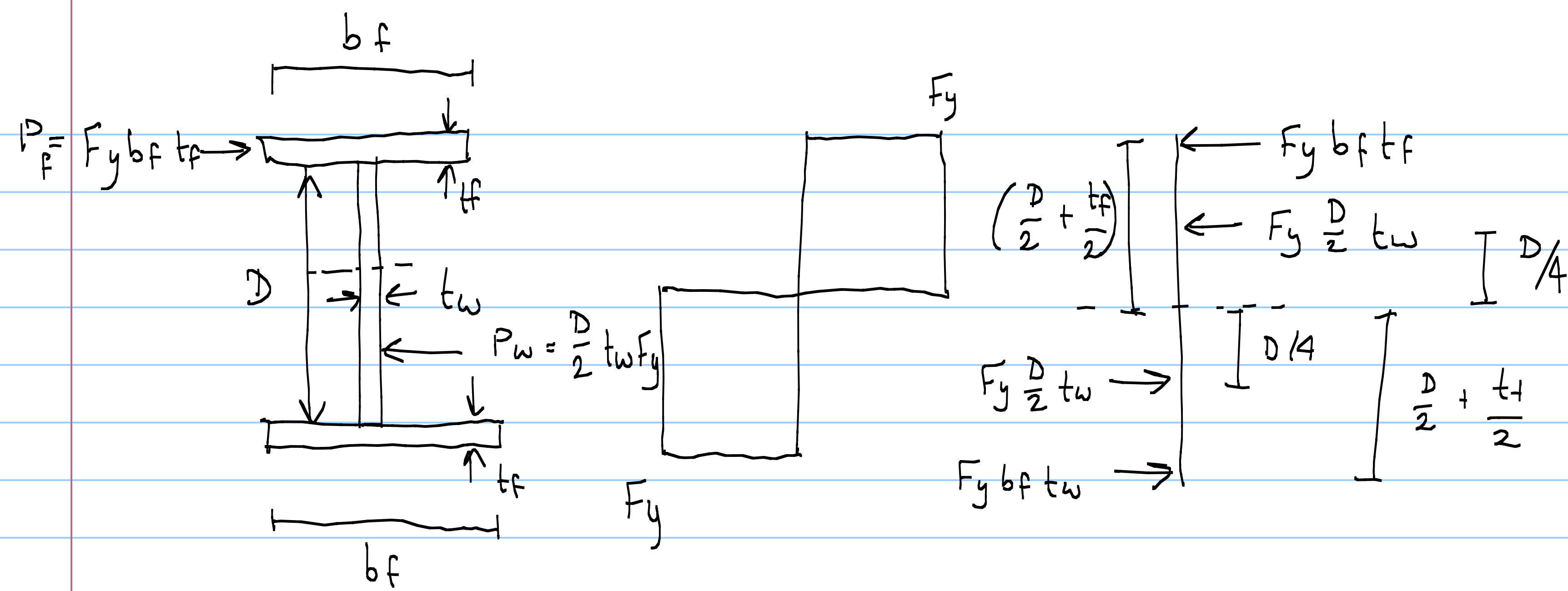
$M_y = F_y S_x$

$S_x = \frac{I}{\bar{y}}$

$M = M_p$ = plastic moment

$M_p = F_y Z_x$

Z_x = plastic section modulus



$$M_p = P \cdot d$$

$$M_p = 2\left(F_y b_f t_f\right)\left(\frac{D}{2} + \frac{t_f}{2}\right) + 2\left(\frac{F_y D t_w}{2}\right)\left(\frac{D}{4}\right)$$

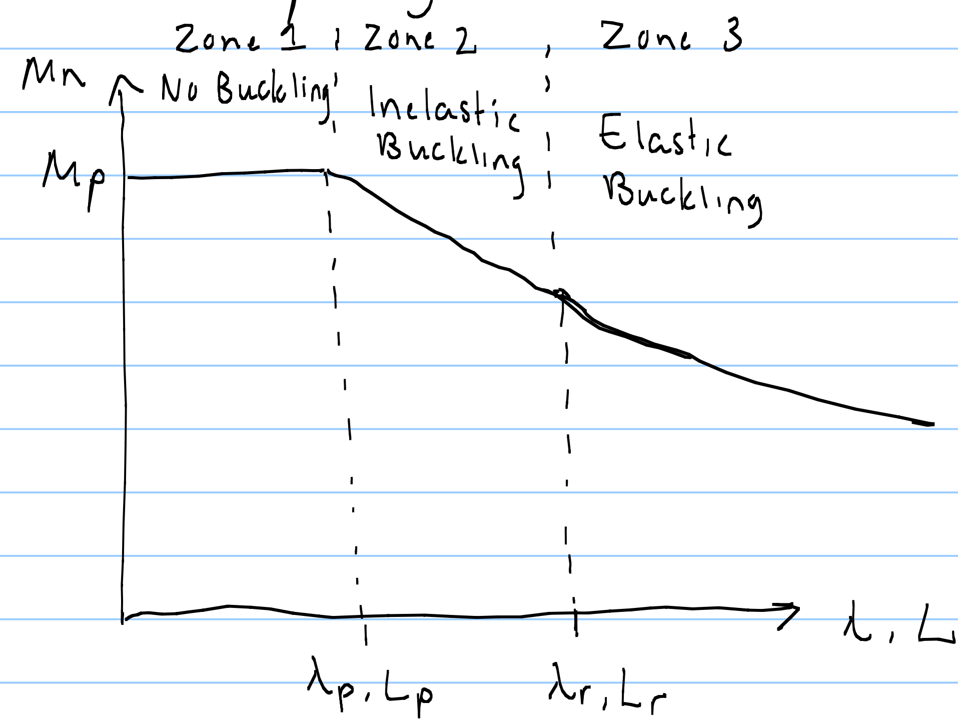
$$M_p = F_y \left[\left(D + t_f\right) b_f t_f + \frac{D^2}{4} t_w \right]$$

$$M_p = F_y Z_x$$

$$Z_x = (D + t_f)\, b_f\, t_f + \frac{D^2}{4} t_w$$

$Z_x \Rightarrow$ given in section database

- Moment Capacity:



Buckling can be prevented if:

$\lambda_f < \lambda_p (f)$

$\lambda_w < \lambda_p (w)$

$L < L_p$

then $\boxed{M_n = M_p}$

Flange Slenderness Limits (Table 5.2)

$$\lambda_p = 0.38\sqrt{E/F_y}$$

$$\lambda_r = 0.83\sqrt{E/F_L} \quad \text{(ksi)}$$

$F_L = F_y - 10$ (ksi) for rolled shapes

↳ reduction for residual stresses

$$\lambda = \frac{b_f}{2t_f}$$

Web Slenderness Limits (Table 5.2)

$$\lambda_p = 3.76\sqrt{E/F_y}$$

$$\lambda_r = 5.70\sqrt{E/F_y}$$

$$\lambda = \frac{h}{t_w}$$

Lateral Bracing Limits

$$L_p = 1.76 r_y \sqrt{E/F_y} \qquad \left(r_y = \sqrt{I_y/A}\right)$$

$$L_r = \frac{r_y X_1}{F_L} \sqrt{1 + \sqrt{1 + X_2 F_L^2}}$$

X_1, X_2 → given in tables

⇒ If: (1) $\frac{b_f}{2t_f} \leq \lambda_p = 0.38\sqrt{E/F_y}$

and (2) $\frac{h}{t_w} \leq \lambda_p = 3.76\sqrt{E/F_y}$

and (3) $L_b \leq L_p$

then $M_n = M_p$ because we will not have buckling

Example: Beam Analysis

Determine the nominal capacity of a W12x22 A36 steel beam with continuous lateral support along its compression flange.

$L_b = 0 < L_p$ ✓

$b_f/2t_f = 4.74 < 0.38\sqrt{29000/36} = 10.8$ ✓

$h/t_w = 41.8 < 3.76\sqrt{29000/36} = 107$ ✓

$\therefore M_n = M_p$

$M_p = F_y Z_x = (36\text{ ksi})(29.3\text{ in}^3) = 1054\text{ in-kip} \approx 87.9\text{ ft-kips}$

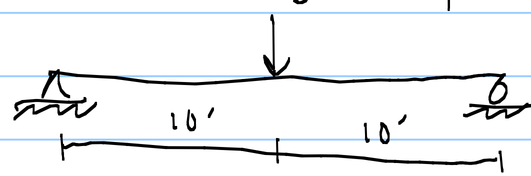
$\phi M_n = \phi M_p \approx 0.9\,(87.9\text{ ft-kips}) = 79.1\text{ ft-kips}$

$\phi = 0.90$

Example: Beam Design

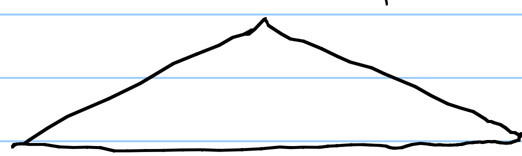
Design A36 beam w/ continuous lateral support:

$P_D = 8$ kips, $P_L = 24$ kips



$P_u = 1.2(8) + 1.6(24) = 48$ kips (neglect beam weight)

$M = \frac{PL}{4}$



$M_u = \frac{P_u L}{4} = \frac{(48 \text{ kips})(20')}{4} = 240$ ft-kips

Assume $\phi M_n = \phi M_p = \phi Z_x F_y \geq M_u = 240$ ft-kips

$$Z_x \geq \frac{M_u}{\phi F_y} = \frac{240 \text{ ft-kips} \, (12 \text{ in/ft})}{(0.9)(36 \text{ ksi})} = 88.9 \text{ in}^3$$

→ from Z_x table, try W21×44.

Check : $\lambda_f < \lambda_p$ ✓ } from table
$\lambda_w < \lambda_p$ ✓ }
$L_b = 0 < L_p$ ✓

Add self weight, $w = 0.044$ k/ft

$$M = \frac{wL^2}{8} = \frac{(0.044)(20)^2}{8} = 2.2 \text{ ft-k}$$

$$M_u = 240 + 1.2(2.2) = 243 \text{ ft-kips}$$

$$\phi M_n = \phi M_p = 0.9(36)(95.8) = 259 \text{ ft-kips} > 243 \checkmark$$

Select W21×44

Summary - Design Procedure for Beams w/ Continuous Lateral Support

1. Compute M_u

2. Calculate required Z_x.

$$Z_x \geq \frac{M_u}{\phi F_y} \quad \text{(may need to convert units)}$$

$$\phi = 0.90 \quad \text{(Assumes } M_n = M_p\text{)}$$

3. Choose preliminary section based on Z_x table.

4. Check flange and web slenderness.

5. If (4) OK, then select trial section
   If (4) not OK, { (i) calculate M_n
   (ii) Choose a new section