Equilibrium

CEIG - 125 Introduction to Civil Engineering

Fall 2005

Lecture 3

Outline

- Free Body Diagrams
- Supports and Reaction Forces
- Equilibrium (Newton's Laws)
- Applying Equilibrium Equations
- Statically Determinate Reactions
- Examples

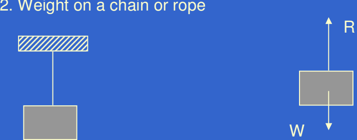
Free Body Diagrams (FBDs)

- Sketch representing a body and forces acting on it.
- Isolates things of interest.
- FBDs are a must for every statics problem being solved
- First thing you do is draw an FBD

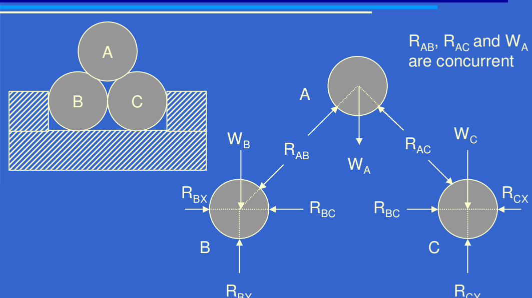
FBDs for Supports

1. Body on an inclined plane

2. Weight on a chain or rope

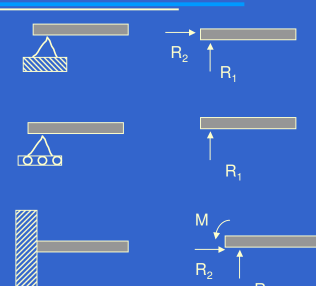


FBD

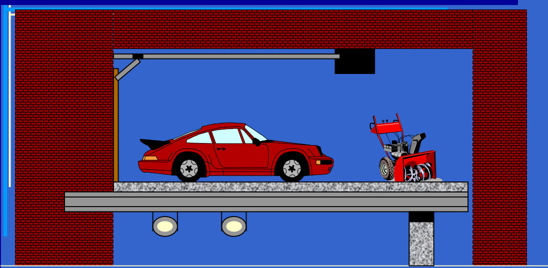


Supports and Reaction Forces

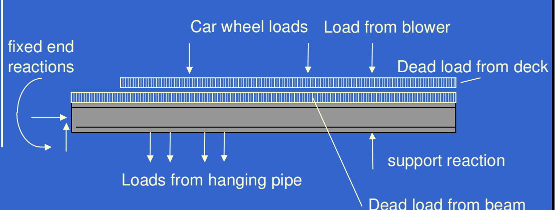
- Pin
- Roller
- Fixed



Complex FBD Example



Complex FBD Example



Equilibrium

- Newton’s Laws:
  - First Law: A particle continues to move in a straight line or remains at rest if there is no unbalanced force
  - Second Law: The acceleration of a particle is proportional to the force acting on it.
  - Third Law: To every action there is an equal and opposite reaction.

Equilibrium

- Law 1 - Defines static equilibrium

  At rest => no unbalanced force

  Σ M = 0 (moments)

  Σ F = 0 (forces)
- Law 2 - States that F $\propto$ a
- Law 3 - Defines interaction between bodies and the rest of the world - reactions

Using Equilibrium

- To determine the magnitude of the reaction forces provided by structure supports:
  - Draw a free body diagram to isolate the structure from its environment and replace all supports with reaction forces
  - Confirm that reactions are statically determinate and structure is stable
  - Generate and solve equlibrium equations

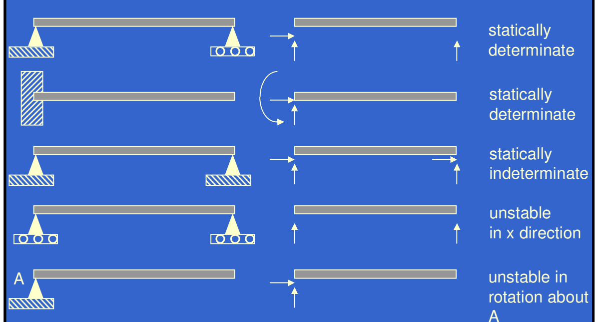
Statically Determinate Reactions

- In two dimensions, we have 3 equilibrium equations (in 3D, 6 equilibrium eqns)
- Statically determinate reactions in 2D means:
  - we have 3 unknown forces
  - we have 3 equilibrium equations

Statically Indeterminate Reactions and Unstable Structures

- If > 3 reactions (6 in 3D), then we have a statically indeterminate set of reactions
- If < 3 reactions (6 in 3D), then we have an unstable structure

Statically Determinate?



Example 1 - Simply Supported Beam

- Consider a 10 m long simply supported beam with a mass of 8 kg/m. The beam supports a mass of 50 kg, 2m from the left end.
- Find the reactions.

Distributed Loads

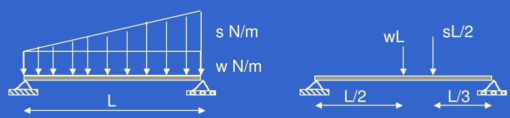
- Extend over an area and along a line.
- Distributed loads can be replaced by a single resultant force acting through a centroid.

Distributed Load Examples

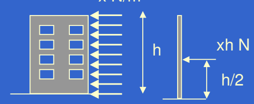
- Dead load of beam

Distributed Load Examples, con't

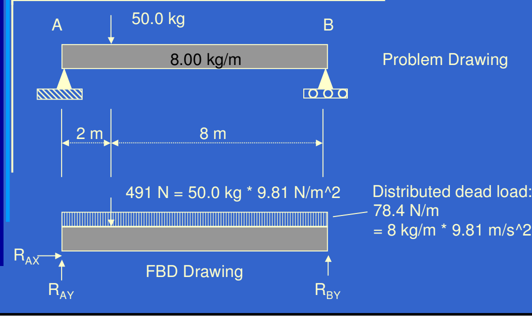
- Drifting snow on a bridge



- Wind on a building



Step 1: Draw FBD for Example 1



Step 2: Statically Determinate?

- We have 3 unknown reactions
- We have 3 equilibrium equations

- Structure's reactions are statically determinate

Step 3: Determine Reactions

$\overset{+}{\rightarrow}\Sigma F_X = 0 \quad = \quad R_{AX}, \quad R_{AX} = 0$

$+\uparrow \Sigma F_Y = 0 \quad = \quad +R_A + R_B - 491\text{ N} - (78.4\text{ N/m} * 10\text{ m})$

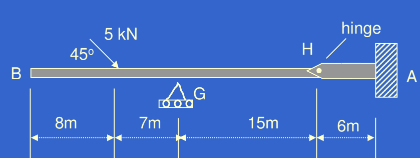
$R_{AY} + R_{BY} \quad = \quad 491\text{ N} + 784 = 1275\text{ N}$

$+\circlearrowleft \Sigma M_A = 0 \quad = \quad +(R_{BY} * 10\text{ m}) - (491\text{ N} * 2\text{m}) - (78.4\text{ N/m} * 10\text{ m} * 5\text{ m})$

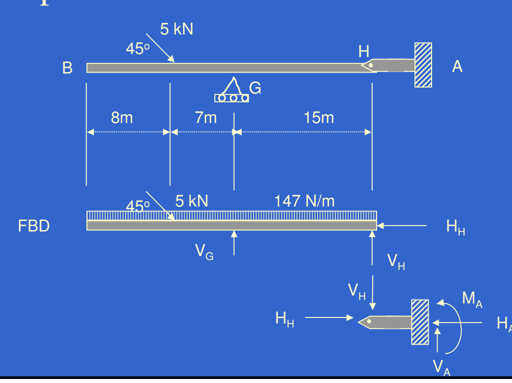
$R_{BY} = 490\text{ N}\uparrow, \quad R_{AY} = 784\text{ N}\uparrow$

Another Harder Example

Given the force system shown below, determine the reactions at G & H. The uniform beam has a mass of 15.0 kg/m.



Step 1: FBD of BH and HA



Step 2: Statically Determinate?

- For section BH, we have 3 unknown reactions: V_H , H_H , and V_G
- We have 3 equilibrium equations
- Section BH's reactions are statically determinate
- Note: after solving section BH, section HA has only 3 unknowns and its reactions can be found using equilibrium

Step 2: Apply Equilibrium to BH

$\overset{+}{\rightarrow} \Sigma F_X \quad = 0 \quad = (5 \text{ kN} \cos 45) - H_H$

$H_H \quad = 3.54 \text{ kN} \leftarrow$

$\overset{+}{\uparrow} \Sigma F_Y \quad = 0 \quad = V_H + V_G - (5 \text{ kN} \cos 45) - 0.147 \text{ kN/m} * 30 \text{ m}$

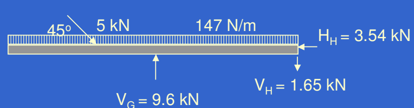
$V_H + V_G = 7.95 \text{ kN}$

$\overset{+}{\circlearrowleft} \Sigma M_H \quad = 0 \quad = +(5 \text{ kN} \cos 45) * 22 \text{ m} -(V_G * 15 \text{ m}) +(0.147 \text{ kN/m}*30\text{m}*15\text{m})$

$V_G = 9.60 \text{ kN} \uparrow \quad , \quad V_H = 1.65 \text{ kN} \downarrow$

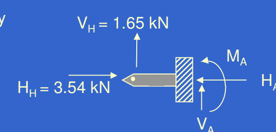
Results for Section BH

FBD



Apply Equilibrium to Section HA

V_H and H_H are applied in equally and oppositely to the way they were applied to section BH.



$\overset{+}{\rightarrow} \Sigma F_X \quad = 0 \quad = 5.54 - H_A$

$H_A \quad = 3.54 \text{ kN} \leftarrow$

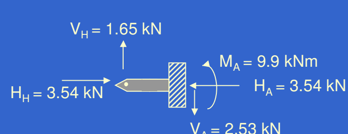
$\overset{+}{\uparrow} \Sigma F_Y \quad = 0 \quad = 1.65 \text{ kN} + V_A - 0.147 \text{ kN/m} * 6 \text{ m}$

$V_A \quad = 0.77 \text{ kN} \uparrow$

$\overset{+}{\circlearrowleft} \Sigma M_A \quad = 0 \quad = -(1.65 \text{ kN} * 6 \text{ m}) + M_A$

$M_A \quad = 9.9 \text{ kNm} = 9.9 \text{ kNm} \circlearrowleft$

Results for Section HA



Summary

- Equilibrium means that no unresisted forces exist
- If we know a structure is in equilibrium, we can use equilibrium equations to find reaction forces on the structure
- What happens if structure is statically indeterminate? What can we do?

Next Week

- Homework 3 is due at the **beginning** of class
- Bridges
- Gunn and Vesilind: Chapter 3, Groups 5 and 6.