Forces and Moments

CIEG-125 Introduction to Civil Engineering

Fall 2005

Lecture 3

Outline

- What is mechanics?
- Scalars and vectors
- Forces are vectors
- Transmissibility of forces
- Resolution of colinear forces
- Moments and couples

What is Mechanics?

- Mechanics (also fluid mechanics, soil mechanics etc) - forces acting on bodies
  - Statics - bodies at rest or moving with uniform velocity
  - Dynamics - bodies accelerating
  - Strength of materials - deformation of bodies under forces
  - Structural Mechanics - focus on behavior of structures under loads

Rigid Body

- Rigid body is a body that ideally does not deform under a force
  - All material deforms
  - When deformations are small assume the body is rigid.
- Examples
  - foam block with a coin
  - wood block with a small weight
  - stone arches

Elastically Deformable Body

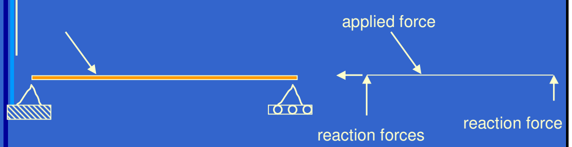
- Bodies that undergo reversible deformations
- Examples:
  - rubber bands
  - springs
  - steel, concrete and wood structures under small deformations
- If structure deforms slightly, we can use original geometry for entire analysis

Inelastically Deformable Body

- Bodies that undergo irreversible deformations due to forces
- Examples:
  - bent paper clip
  - steel, concrete and wood structures under large deformations
- If a structure exhibits large elastic or inelastic deformations, geometry changes

Statics

- We start with statics
- Determining that the various forces acting on a body are in equilibrium.

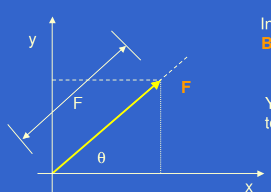


Forces

- Are actions of one body on another
- Pushing against each other -
  - compressive force (bodies in compression)
- Pulling against each other -
  - tensile force (bodies in tension)
- Forces represented by arrow
  - length of arrow = scalar magnitude
  - direction of arrow = line of action of force

Forces

- Force is a vector quantity



In these lecture notes, we will use **Boldface** to represent a vector.

E.g., **F** is a vector

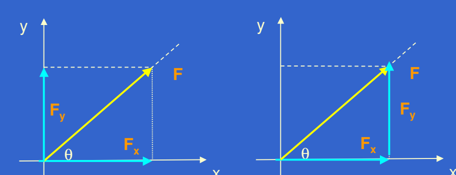
Your book uses arrows over letters to signify a vector (hard to see).

E.g., $\vec{F}$ is a vector

In these lecture notes, we will use F to represent the magnitude of **F** . Your book uses |F| or F to signify the magnitude of **F** .

Force Components

- Force can be replaced by x and y components



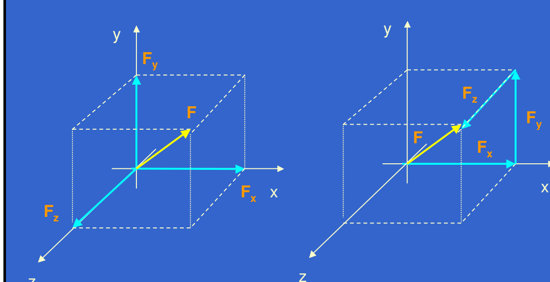
$F_x = F \cos\theta$
$F_y = F \sin\theta$

Forces in 3 Dimensions

- In 3 dimensions, forces can be replaced by 3 components along x, y and z axes:

$\mathbf{F} = (F_x, F_y, F_z) = (F_1, F_2, F_3)$ or
$\mathbf{F} = F_1\,\mathbf{i} + F_2\,\mathbf{j} + F_3\,\mathbf{k}$

i, **j** and **k** are unit vectors in **x**, **y**, and **z**



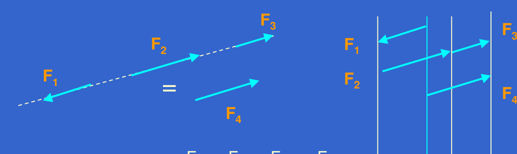
$F = (F_x^2 + F_y^2 + F_z^2)^{0.5}$

Types of Forces

- Colinear
- Concurrent
- Coplanar

Colinear Forces

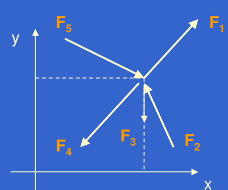
- Forces acting along the same line of action.
- The magnitude of a single equivalent force is the same as the sum of the colinear forces



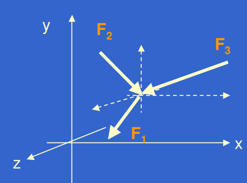
$$-F_1 + F_2 + F_3 = F_4$$

Concurrent Forces

- Pass through the same point in space



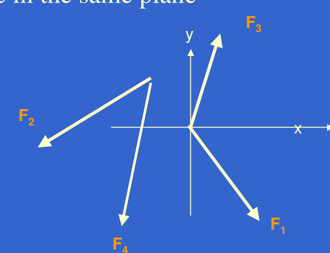
2D Case



3D case

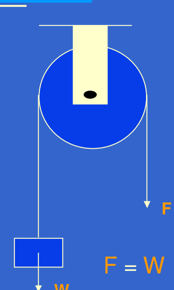
Co-planar Forces

- Lie in the same plane



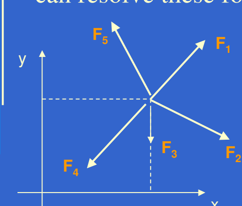
Transmissibility

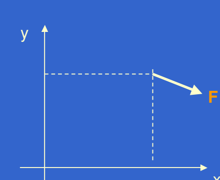
- Extension of the concept of colinear forces.
- If a force is exerted on a rope or a cable, then each end must have an equal force if the system does not move.



Resolution of Forces

- If we have a set of concurrent forces, we can resolve these forces into a single force.

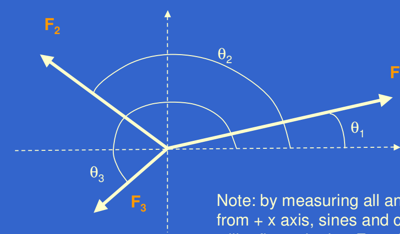




Resolution of Forces .... con't.

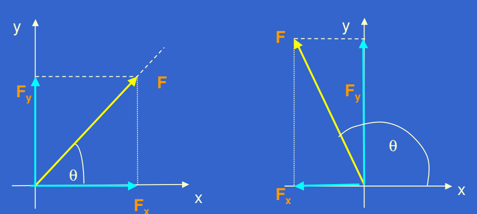
- To compute the resultant of several concurrent forces:
  - first determine the angle of each force with respect to + x axis
  - find x and y components for each force; and
  - sum colinear forces
- Note: You must be systematic about the angles. I will show you one system.

Determine angles wrt +X axis



Note: by measuring all angles from + x axis, sines and cosines will reflect whether F_x and F_x are positive or negative.

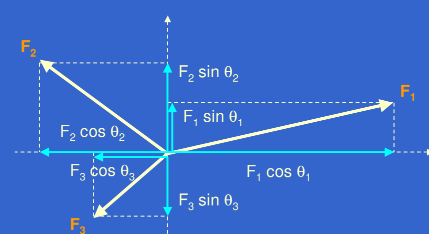
Determine Force Components



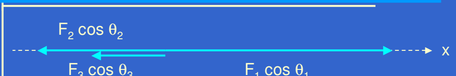
$F_y = F \sin\theta$ (+)
$F_x = F \cos\theta$ (+)

$F_y = F \sin\theta$ (+)
$F_x = F \cos\theta$ (–)

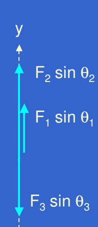
Determine X and Y components for each force



Sum colinear forces in x and y



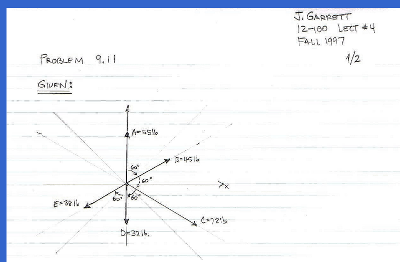
$F_x = F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3$
$F_Y = F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3$



Note: all forces are summed because the sines and cosines will indicate whether the component is in the positive or negative direction.

Determine Magnitude and Direction of Resultant Force

- Then compute the resultant force:
  - magnitude and $F = (F_x^2 + F_Y^2)^{0.5}$
  - direction $\theta = \tan^{-1}(F_Y / F_x)$



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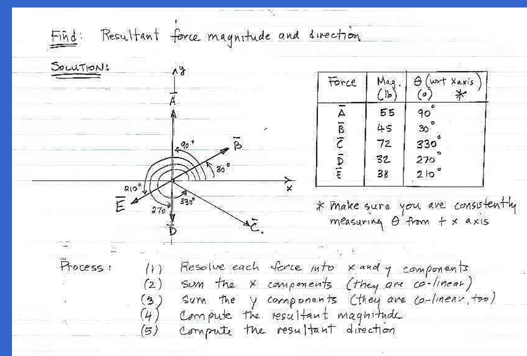
PROBLEM 9.11

GIVEN:



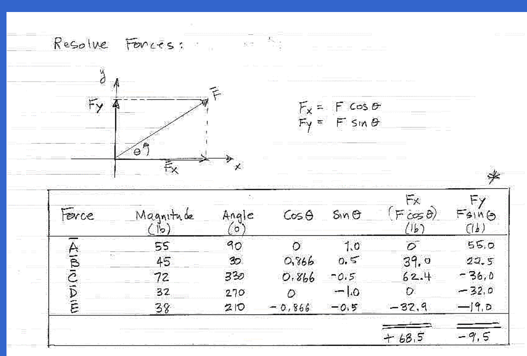
Find: Resultant force magnitude and direction

SOLUTION:

Force	Mag. (lb)	θ (wrt x axis) (°) *
A	55	90°
B	45	30°
C	72	330°
D	32	270°
E	38	210°

* make sure you are consistently measuring θ from + x axis

Process:
(1) Resolve each force into x and y components
(2) sum the x components (they are co-linear)
(3) sum the y components (they are co-linear, too)
(4) Compute the resultant magnitude
(5) Compute the resultant direction

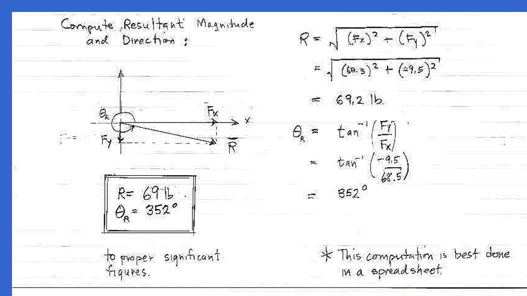


Resolve Forces:

$F_x = F \cos\theta$
$F_y = F \sin\theta$

Force	Magnitude (lb)	Angle (°)	Cos θ	Sin θ	Fx (F cos θ) (lb)	Fy F sin θ (lb)
A	55	90	0	1.0	0	55.0
B	45	30	0.866	0.5	39.0	22.5
C	72	330	0.866	-0.5	62.4	-36.0
D	32	270	0	-1.0	0	-32.0
E	38	210	-0.866	-0.5	-32.9	-19.0
					+ 68.5	-9.5

*



Compute Resultant Magnitude and Direction:

$R = \sqrt{(F_x)^2 + (F_y)^2}$

$= \sqrt{(68.5)^2 + (-9.5)^2}$

$= 69.2$ lb.

$\theta_R = \tan^{-1}\left(\frac{F_y}{F_x}\right)$

$= \tan^{-1}\left(\frac{-9.5}{68.5}\right)$

$= 352°$

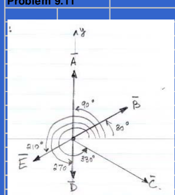
R = 69 lb.
θ_R = 352°

to proper significant figures.

* This computation is best done in a spreadsheet.

Spreadsheet Solution

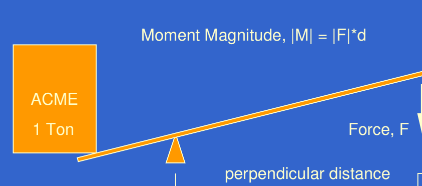
Problem 9.11



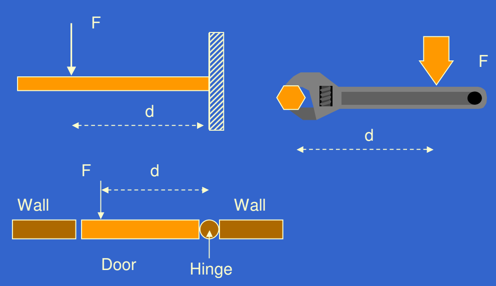
Force	Mag	Theta	Sin	Cos	Fx Fcos(theta)	Fy Fsin(theta)
	lb	deg			lb	lb
A	55	90	1	0.000	0.0	55.0
B	45	30	0.5	0.866	39.0	22.5
C	72	330	-0.5	0.866	62.4	-36.0
D	32	270	-1	0.000	0.0	-32.0
E	38	210	-0.5	-0.866	-32.9	-19.0
					68.4	-9.5
Resultant Magnitude of Force =					69.1	lbs
Angle of the Resultant Force =					-7.9	degrees

Moments and Couples

- A moment about a point is defined as the product of a force magnitude and the perpendicular distance from the force line of action to that point.



Moment Examples

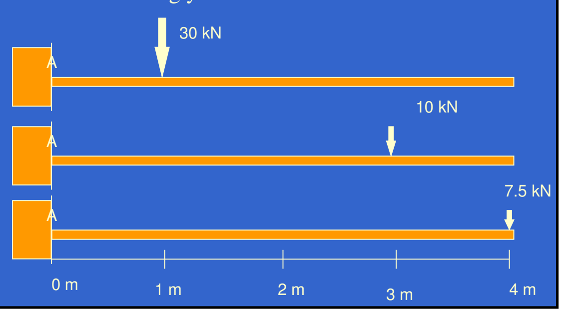


Moments, con't.

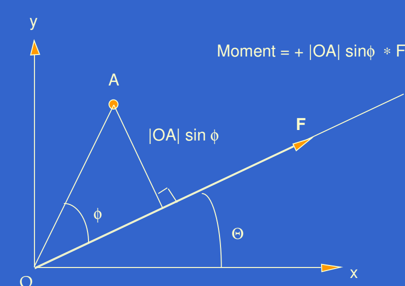
- Moments exist even when rotation is being resisted
- By convention, counter clockwise moments are positive
- The total moment about a point is the sum of the individual moments

Moments, con't.

- The following yield identical moments about A:



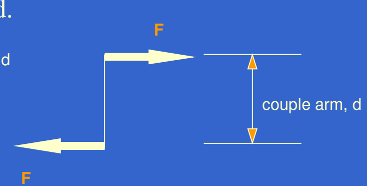
Example



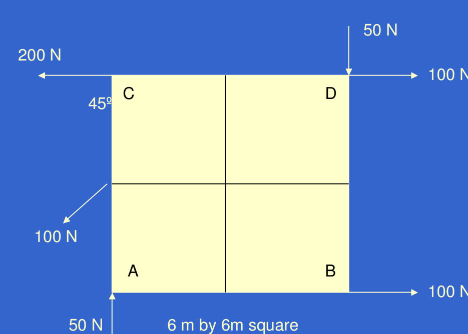
Couples

- Couples are pairs of forces acting in opposite directions and separated by a distance d.

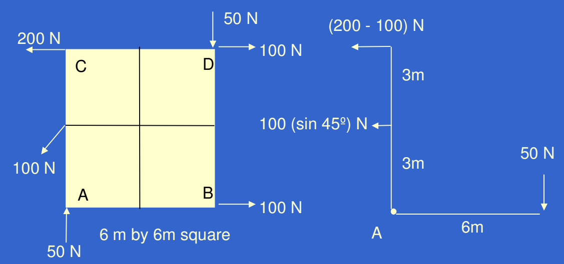
Moment = F * d



What is the total moment about corner A?



What is the total moment about corner A?



Moment = (200 - 100 N) * 6m + (100 sin 45º N) * 3m - (50 N) * 6m = 371 Nm

Summary

- Forces are vectors.
- To "add" forces:
  - you must decompose into components
  - add magnitudes of colinear components
  - called resolution
- Moments are caused by forces acting at a perpendicular distance from a point