EXAM 1

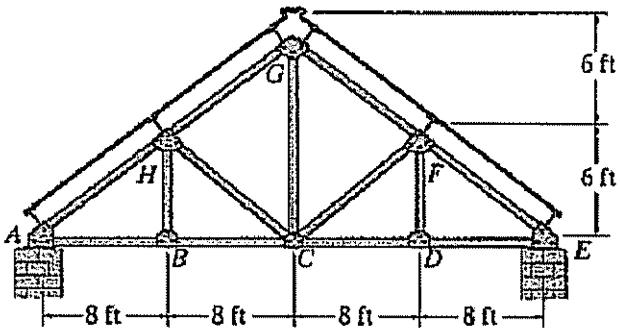
CIEG 301: STRUCTURAL ANALYSIS
FALL 2006

NAME Solution

	Score
Problem 1	
Problem 2	
Problem 3	
Extra Credit	
Total	

PROBLEM 1 [30 points]

The roof trusses shown in the figure below are uniformly spaced at 15 feet. The roofing material and purlins have an average combined weight of 6 lb/ft². The building is located in New York where the anticipated snow load is 20 lb/ft². These loadings occur over the horizontal projected area of the roof. Determine the force in members **AB, HC and FD using the method of joints for member AB and the method of sections for member HC.** Ignore the self weight of the truss and all other loads other than those given. Assume the support at A acts as a pin and the support at E acts as a roller. **Hint:** First determine the governing load combination.



ASCE Load Combinations:

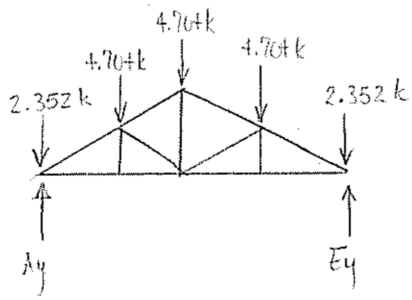
- 1.4D
- 1.2D + 1.6L + 0.5*max(Lr, S, or R)
- 1.2D + 1.6*max(Lr, S, or R) + max(0.5L, 0.8W)
- 1.2D + 1.6W + 0.5L + 0.5*max(Lr, S, or R)
- 1.2D + 1.0E + 0.5L + 0.2S
- 0.9D ± 1.6W
- 0.9D ± 1.0E

Where: D = dead load, L=live load, Lr = roof live load, S=snow load, R=rain load, W=wind load, and E = seismic load

- Relevant load combinations (by inspection of one these two will govern):

  $1.4D = 1.4(6) = 8.4$ psf

  $1.2D + 1.6S = 1.2(6) + 1.6(20) = \underline{39.2 \text{ psf}}$ (governs)



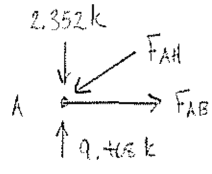
- Tributary area: 15′ × 4′ for joints A and E = 60 ft²

  15′ × 8′ for joints F, G, and H = 120 ft²

- Joint loads: A and E : (60 ft²)(39.2 psf) = 2.352 k

  F, G, and H : (120 ft²)(39.2 psf) = 4.704 k

- Reactions: Symmetric structure ∴ $A_y = E_y = \frac{1}{2}\left[3(4.704) + 2(2.352)\right] = 9.408$ k

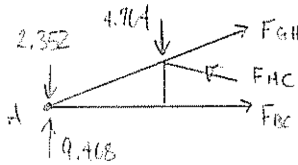


- Method of joints for AB: $\sum F_y = 9.408 - 2.352 - \frac{3}{5} F_{AH} = 0$

  $F_{AH} = 11.76$ k (C)

  $\sum F_x = -\frac{4}{5}(11.76) + F_{AB} = 0$

  $\boxed{F_{AB} = 9.408 \text{ k (T)}}$



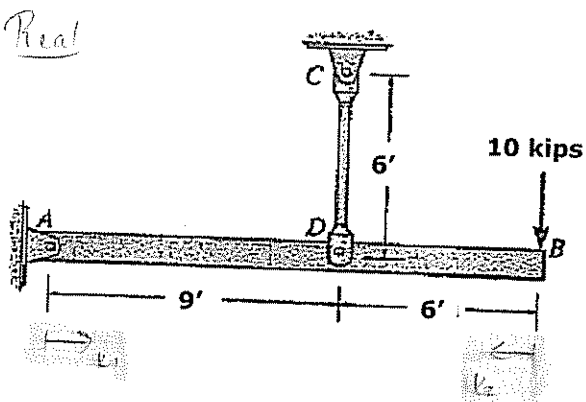
- Method of sections for HC: $\sum M_A = 4.704(8) - (3/5)(F_{HC})(8) - (4/5)(F_{HC})(6) = 0$

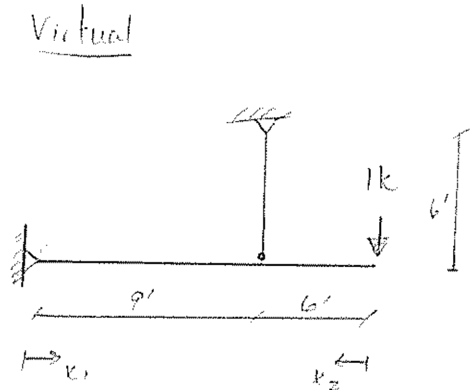
  $\boxed{F_{HC} = 3.92 \text{ (C)}}$

PROBLEM 2

[35 points]

Determine the displacement at point B in the structure shown below **using virtual work.** Note the following: Member CD is connected to Member AB via a pin connection at point D; Member AB has a moment of inertia of 1200 in^4 and an area of 18.00 in^2; Member CD has a moment of inertia of 0.750 in^4 and an area of 3.00 in^2; and both members are made of steel with E = 29,000 ksi.





$U_E = U_I$

$$(1k)(\Delta) = \int_A^D \frac{Mm}{EI} + \int_D^B \frac{Mm}{EI} + \frac{NnL}{AE}\Big|_{CD}$$

Real system: $\Sigma M_A = (10k)(15') - F_{CD}(9') = 0$

$F_{CD} = 16.67 \text{ k (T)}$

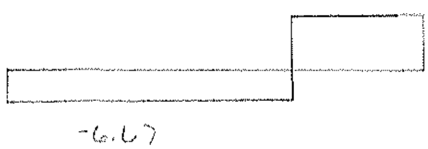
ΣF_y: $A_y = 16.67 - 10 = 6.67 \text{ k} \downarrow$

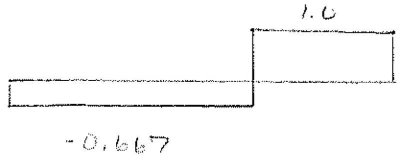
Virtual system: All forces are 1/10 real forces

$F_{CD} = 1.667 \text{ k (T)}$

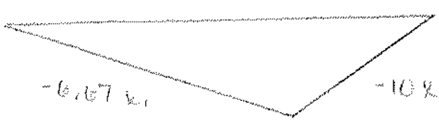
Member AB:

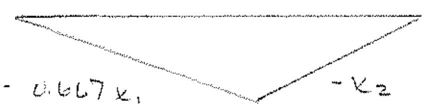
V





M





$$\Delta = \left[\int_0^9 \frac{(-6.67x_1)(-0.667x_1)}{EI} + \int_0^6 \frac{(-10x_2)(-x_2)}{EI} + \frac{(16.67)(1.67)(6)}{AE}\right] / 1k$$

$$\Delta = \left[\frac{1}{EI}\left(1.482x_1^3\Big|_0^9 + 3.33x_2^3\Big|_0^6\right) + \frac{1}{AE}(167.03)\right] / 1k$$

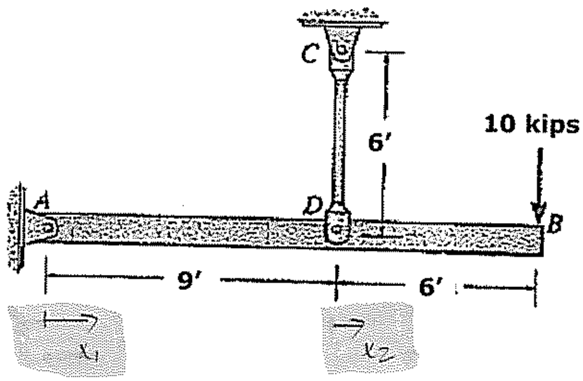
$$\Delta = \left[\frac{(1080 + 720)\,k^2\,ft^3\,\left(\frac{12\text{ in}}{ft}\right)^3}{(29000\text{ ksi})(1200\text{ in}^4)} + \frac{(167.03)\,k^2\,ft\,\left(\frac{12\text{ in}}{ft}\right)}{(29000\text{ ksi})(3\text{ in}^2)}\right] / 1k$$

$$\boxed{\Delta = 0.112'' \downarrow}$$

PROBLEM 2 [35 points]

Determine the displacement at point B in the structure shown below **using virtual work.** Note the following: Member CD is connected to Member AB via a pin connection at point D; Member AB has a moment of inertia of 1200 in^4 and an area of 18.00 in^2; Member CD has a moment of inertia of 0.750 in^4 and an area of 3.00 in^2; and both members are made of steel with E = 29,000 ksi.

if coordinate system defined as shown, moment eqns and related calcs change to:



Real: $M(x_1) = -6.67x_1$

$M(x_2) = -60 + 10x_2$

Virtual: $m(x_1) = -0.667x_1$

$m(x_2) = -6 + x_2$

$$\Delta = \left[\int_0^9 \frac{(-6.67x_1)(-0.667x_1)}{EI} + \int_0^6 \frac{(-60+10x_2)(-6+x_2)}{EI} + \frac{(16.67)(1.667)}{AE}\right] / 1k$$

$$\Delta = \left[\frac{1}{EI}(1.482\,x_1^3)\Big|_0^9 + \frac{1}{EI}\left(360x_2 - 60x_2^2 + \frac{10}{3}x_2^3\right)\Big|_0^6 + \frac{1}{AE}(167.03)\right] / 1k$$

$$\Delta = \left[\frac{(1080+720)k^2 ft^3 (12 in/ft)^3}{(29000\ ksi)(1200\ in^4)} + \frac{(167.03)k^2 ft\,(12\ in/ft)}{(29000\ ksi)(3\ in^2)}\right] / 1k$$

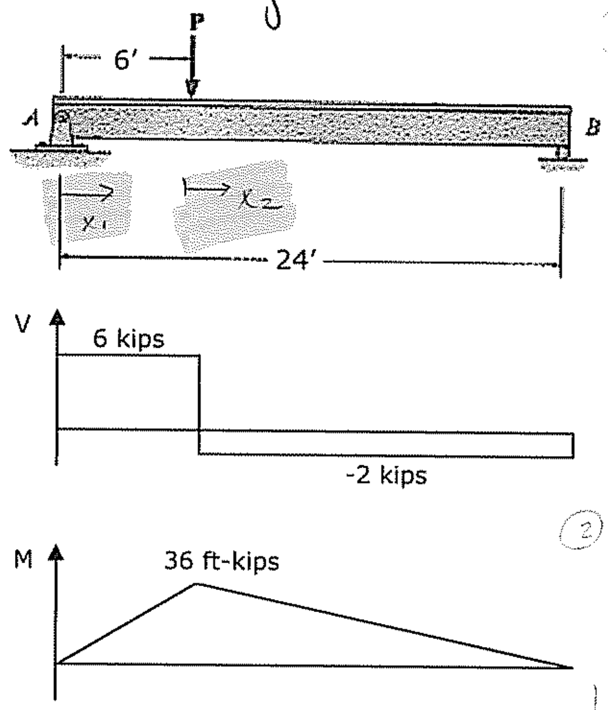
$$\boxed{\Delta = 0.112'' \downarrow}$$

PROBLEM 3 35 points

(a) Determine the maximum displacement in the beam shown below **as a function of EI**. P is equal to 8 kips, resulting in the shear and moment diagrams shown below.

(b) If E is equal to 29,000 ksi and the maximum allowable displacement is 1.5", **what values of I would render an acceptable design for this member?**

for the following coordinates:



(a) Integrate moment eqns.

① • $M(x_1) = 6x_1$

$$v(x_1) = \frac{1}{EI}\iint 6x_1\,dx_1 = \frac{1}{EI}\int 3x_1^2 + C_1$$

$$= \frac{1}{EI}\left(x_1^3 + C_1x_1 + C_2\right)$$

• $M(x_2) = 36 - 2x_2$

$$v(x_2) = \frac{1}{EI}\iint 36 - 2x_2\,dx_2 = \frac{1}{EI}\int 36x_2 - x_2^2 + C_3\,dx_2$$

$$= \frac{1}{EI}\left(-\frac{1}{3}x_2^3 + 18x_2^2 + C_3x_2 + C_4\right)$$

② Apply B.C.s.

• $v(x_1 = 0) = 0 = \frac{1}{EI}\left(0^3 + 0 + C_2\right)$

$\therefore C_2 = 0$

• $v(x_2 = 18) = 0 = \frac{1}{EI}\left(-\frac{1}{3}(18)^3 + 18^3 + C_3(18) + C_4\right)$

$0 = \frac{1}{EI}\left(3888 + 18C_3 + C_4\right)$

$\boxed{1}$ $18C_3 + C_4 = -3888$

③ Apply continuity conditions

• $\frac{dv_1}{dx_1}(x_1 = 6) = \frac{dv_2}{dx_2}(x_2 = 0)$

$3(6)^2 + C_1 = C_3$

$\boxed{2}$ $C_3 - C_1 = 108$

$v_1(x_1 = 6) = v_2(x_2 = 0)$

$6^3 + 6C_1 + C_2 = C_4$

$\boxed{3}$ $6C_1 + \cancel{C_2}^{0} - C_4 = -216$

④ Solve simultaneous eqns:

$\boxed{2}$ $C_3 = 108 + C_1$

$\boxed{1}$ $18(108 + C_1) + C_4 = -3888$

$18C_1 + C_4 = -5832$

$\boxed{3}$ $6C_1 - C_4 = -216$

$24C_1 = -6048$

$C_1 = -252$

$C_3 = 108 - 252 = -144$

$C_4 = -1296$

⑤ Find max v:

Assume v_{max} occurs in x_2

$$\frac{dv_2}{dx_2} = 36x_2 - x_2^2 - 144 = 0$$

$x_2 = 4.584$ or 31.42 (not valid)

$\therefore v_{max}$ does occur in x_2

$$v_2(x_2 = 4.584') = \frac{1}{EI}\left(\frac{-(4.584)^3}{3} + 18(4.584)^2 - 144(4.584) - 1296\right)$$

(b) $\frac{2.782(10)^3}{(29000)(1.5)} \leq I$

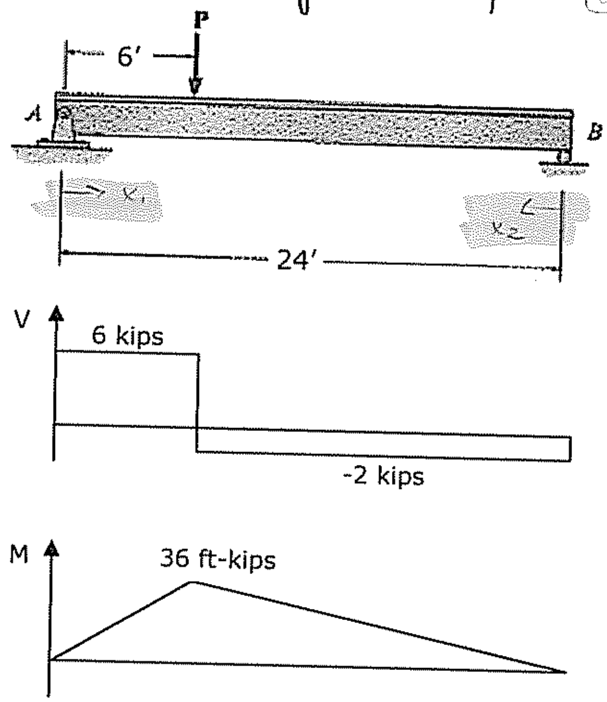
PROBLEM 3

[35 points]

(a) Determine the maximum displacement in the beam shown below **as a function of EI**. P is equal to 8 kips, resulting in the shear and moment diagrams shown below.

(b) If E is equal to 29,000 ksi and the maximum allowable displacement is 1.5", **what values of I would render an acceptable design for this member?**

or, for the following coordinate system:



(a) 1. Integrate moment equations

$$M(x_1) = 6x_1$$

$$EI\,v(x_1) = \iint 6x_1\,dx_1 = \int 3x_1^2 + C_1\,dx_1 = x_1^3 + C_1x_1 + C_2$$

$$M(x_2) = 2x_2$$

$$EI\,v(x_2) = \iint 2x_2\,dx_2 = \int x_2^2 + C_3\,dx_2 = \tfrac{1}{3}x_2^3 + C_3x_2 + C_4$$

2. Apply boundary conditions

$$EI\,v_1(x_1=0) = 0 = 0 + 0 + C_2 \quad \therefore C_2 = 0$$

$$EI\,v_2(x_2=0) = 0 = 0 + 0 + C_4 \quad \therefore C_4 = 0$$

3. Apply continuity conditions

$$v_1(x_1=6) = v_2(x_2=18)$$

$$(6)^3 + C_1(6) = \tfrac{1}{3}(18)^3 + C_3(18)$$

$$C_1 - 3C_3 = 288$$

$$-\frac{dv_1}{dx_1}(x_1=6) = \frac{dv_2}{dx_2}(x_2=18)$$

$$-3(36) - C_1 = 18^2 + C_3$$

$$-432 = C_3 + C_1$$

4. Solve simultaneous equations for C_1 and C_3

$$C_1 - 3C_3 = 288$$

$$-[C_1 + C_3 = -432]$$

$$-4C_3 = 720$$

$$C_3 = -180$$

$$C_1 = -432 + 180 = -252$$

5. Solve for v_{max}

Assume v_{max} occurs in x_2:

$$\frac{dv_2}{dx_2} = x_2^2 - 180 = 0$$

$$x_2 = 13.416'$$

$$v_2(x_2 = 13.416) = \frac{1}{EI}\left[\tfrac{1}{3}(13.416)^3 - 180(13.416)\right]$$

$$\boxed{v_{2\text{-max}} = \frac{-1610\text{ kft}^3}{EI} = \frac{-2.782(10)^3\text{ kin}^3}{EI} = \downarrow}$$

(b)

$$\frac{2.782(10)^3\text{ kin}^3}{(29000\text{ ksi})(I)} \leq 1.5''$$

$$I \geq \frac{2.782(10^3)\text{ k}\cdot\text{in}^3}{(29000\text{ ksi})(1.5\text{ in})} = \boxed{64\text{ in}^4}$$

Extra Credit

1. In addition to strength, **list** three other factors a structural engineer must consider in his / her design? [3 pts]

   Serviceability (deflection etc)
   Maintainability
   aesthetics
   economy
   Constructibility

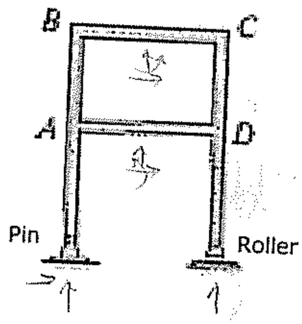
2. **List** three examples of environmental loads that must be considered in structural engineering. [3 pts]

   Snow
   Wind
   Earthquake / Seismic

   Also ok : rain, ice, temperature, ... other relevant bridge loadings

3. Which of the following structures can be analyzed using statics? **Explain** your answers. [4 pts]

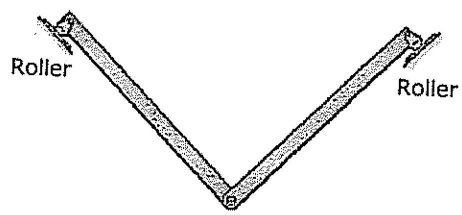
(a)



NO

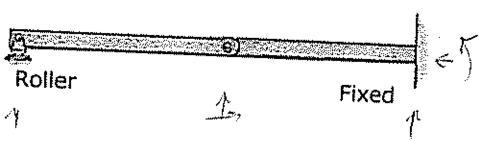
$r = 9$
$3n = 6$ ∴ Statically indeterminant to 3rd degree

(b)



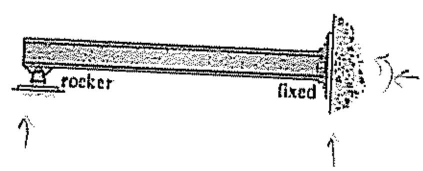
No - not stable

(c)



$r = 6$
$3n = 6$ ∴ yes. statically determinant and stable

(d)



$r = 4$
$3n = 3$

∴ statically indeterminant to 1st degree