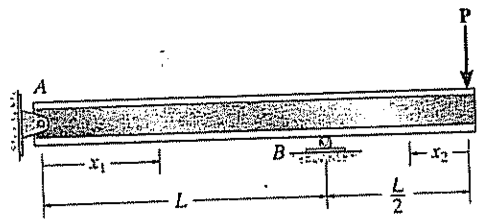
*8-4. Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. Specify the beam's maximum deflection. EI is constant.



Support Reactions and Elastic Curve : As shown on FBD(a).

Moment Function : As shown on FBD(b) and (c).

Slope and Elastic Curve :

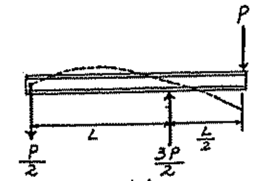
$$EI\frac{d^2v}{dx^2}=M(x)$$

For $M(x_1)=-\frac{P}{2}x_1$,

$$EI\frac{d^2v_1}{dx_1^2}=-\frac{P}{2}x_1$$

$$EI\frac{dv_1}{dx_1}=-\frac{P}{4}x_1^2+C_1 \qquad [1]$$

$$EI\,v_1=-\frac{P}{12}x_1^3+C_1x_1+C_2 \qquad [2]$$



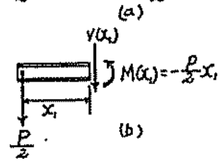
(a)

For $M(x_2)=-Px_2$,

$$EI\frac{d^2v_2}{dx_2^2}=-Px_2$$

$$EI\frac{dv_2}{dx_2}=-\frac{P}{2}x_2^2+C_3 \qquad [3]$$

$$EI\,v_2=-\frac{P}{6}x_2^3+C_3x_2+C_4 \qquad [4]$$



(b)

Boundary Conditions :

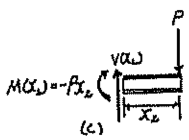
$v_1=0$ at $x_1=0$. From Eq.[2], $C_2=0$

$v_1=0$ at $x_1=L$. From Eq.[2],

$$0=-\frac{PL^3}{12}+C_1L \qquad C_1=\frac{PL^2}{12}$$

$v_2=0$ at $x_2=\frac{L}{2}$. From Eq. [4],

$$0=-\frac{PL^3}{48}+\frac{L}{2}C_3+C_4 \qquad [5]$$



(c)

Continuity Conditions :

At $x_1=L$ and $x_2=\frac{L}{2}$, $\frac{dv_1}{dx_1}=-\frac{dv_2}{dx_2}$. From Eqs.[1] and [3],

$$-\frac{PL^2}{4}+\frac{PL^2}{12}=-\left(-\frac{PL^2}{8}+C_3\right) \qquad C_3=\frac{7PL^2}{24}$$

From Eq.[5], $C_4=-\frac{PL^3}{8}$

The Slope : Substitute the value of C_1 into Eq.[1],

$$\frac{dv_1}{dx_1}=\frac{P}{12EI}\left(L^2-3x_1^2\right)$$

$$\frac{dv_1}{dx_1}=0=\frac{P}{12EI}\left(L^2-3x_1^2\right) \qquad x_1=\frac{L}{\sqrt{3}}$$

The Elastic Curve : Substitute the values of C_1, C_2, C_3, and C_4 into Eqs.[2] and [4], respectively.

$$v_1=\frac{Px_1}{12EI}\left(-x_1^2+L^2\right) \qquad \text{Ans}$$

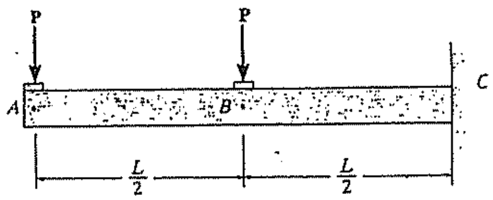
$$v_D=v_1\big|_{x_1=\frac{L}{\sqrt{3}}}=\frac{P\left(\frac{L}{\sqrt{3}}\right)}{12EI}\left(-\frac{L^2}{3}+L^2\right)=\frac{0.0321PL^3}{EI}$$

$$v_2=\frac{P}{24EI}\left(-4x_2^3+7L^2x_2-3L^3\right) \qquad \text{Ans}$$

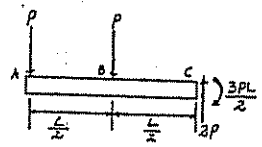
$$v_C=v_2\big|_{x_2=0}=-\frac{PL^3}{8EI}$$

Hence, $v_{max}=v_C=\frac{PL^3}{8EI}$ Ans

8-11. The beam is subjected to the two loads. Use the moment-area theorems and determine the slope and displacement at points A and B. EI is constant.



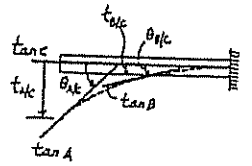
Moment-Area Theorems : The slope at support C is zero. The slopes at A and B are,



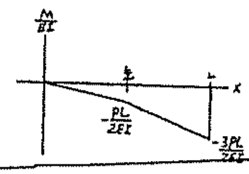
$$\theta_A = |\theta_{A/C}| = \frac{1}{2}\left(-\frac{PL}{2EI}\right)\left(\frac{L}{2}\right)+\left(-\frac{PL}{2EI}\right)\left(\frac{L}{2}\right)+\frac{1}{2}\left(-\frac{PL}{EI}\right)\left(\frac{L}{2}\right)$$
$$= \frac{5PL^2}{8EI} \qquad \textbf{Ans}$$

$$\theta_B = |\theta_{B/C}| = \left(-\frac{PL}{2EI}\right)\left(\frac{L}{2}\right)+\frac{1}{2}\left(-\frac{PL}{EI}\right)\left(\frac{L}{2}\right) = \frac{PL^2}{2EI} \qquad \textbf{Ans}$$

The displacements at A and B are,

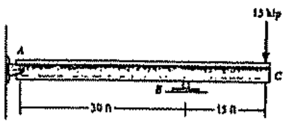


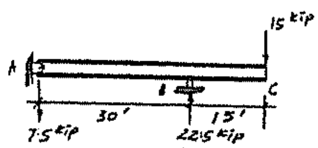
$$\Delta_A = |t_{A/C}| = \frac{1}{2}\left(-\frac{PL}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right)+\left(-\frac{PL}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2}+\frac{L}{4}\right)$$
$$+\frac{1}{2}\left(-\frac{PL}{EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2}+\frac{L}{3}\right)$$
$$= \frac{7PL^3}{16EI} \downarrow \qquad \textbf{Ans}$$

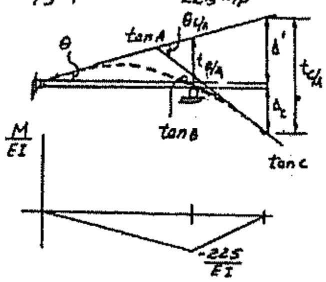


$$\Delta_B = |t_{B/C}| = \left(-\frac{PL}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{4}\right)+\frac{1}{2}\left(-\frac{PL}{EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right)$$
$$= \frac{7PL^3}{48EI} \downarrow \qquad \textbf{Ans}$$

8-12. Use the moment-area theorems to determine the slope and deflection at C. EI is constant.







$$\theta_A = \frac{|t_{B/A}|}{30}$$
$$t_{B/A} = \frac{1}{2}\left(\frac{-225}{EI}\right)(30)(10) = \frac{-33\,750}{EI}$$
$$\theta_A = \frac{1125}{EI}$$

$$\theta_{C/A} = \frac{1}{2}\left(\frac{-225}{EI}\right)(30)+\frac{1}{2}\left(\frac{-225}{EI}\right)(15) = \frac{-5062.5}{EI} = \frac{5062.5}{EI}$$

$$\theta_C = \theta_{C/A} + \theta_A$$
$$\theta_C = \frac{5062.5}{EI} - \frac{1125}{EI} = \frac{3937.5}{EI} \qquad \textbf{Ans}$$

$$\Delta_C = |t_{C/A}| - \frac{45}{30}|t_{B/A}|$$

$$t_{C/A} = \frac{1}{2}\left(-\frac{225}{EI}\right)(30)(25)+\frac{1}{2}\left(-\frac{225}{EI}\right)(15)(10) = -\frac{101\,250}{EI}$$

$$\Delta_C = \frac{101\,250}{EI} - \frac{45}{30}\left(\frac{33\,750}{EI}\right) = \frac{50\,625}{EI} \qquad \textbf{Ans}$$