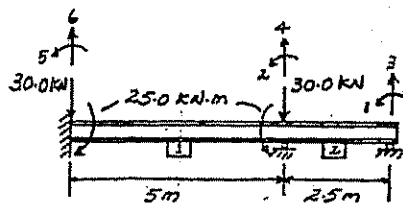
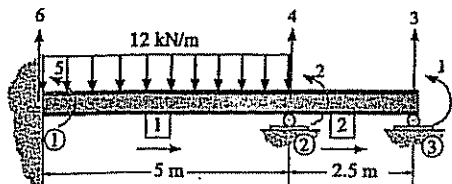


15-4. Determine the reactions at the supports. EI is constant.

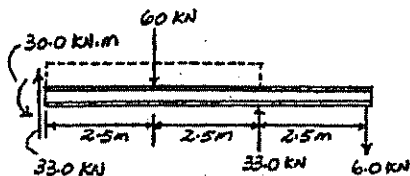


Member 1

$$k_1 = EI \begin{bmatrix} 0.096 & 0.24 & -0.096 & 0.24 \\ 0.24 & 0.8 & -0.24 & 0.4 \\ -0.096 & -0.24 & 0.096 & -0.24 \\ 0.24 & 0.4 & -0.24 & 0.8 \end{bmatrix}$$

Member 2

$$k_2 = EI \begin{bmatrix} 0.768 & 0.96 & -0.768 & 0.96 \\ 0.96 & 1.6 & -0.96 & 0.8 \\ -0.768 & -0.96 & 0.768 & -0.96 \\ 0.96 & 0.8 & -0.96 & 1.6 \end{bmatrix}$$



$$\begin{bmatrix} 0 \\ 25.0 \\ Q_2 \\ Q_2 - 30.0 \\ Q_3 - 25.0 \\ Q_3 - 30.0 \end{bmatrix} = EI \begin{bmatrix} 1.6 & 0.8 & -0.96 & 0.96 & 0 & 0 \\ 0.8 & 2.4 & -0.96 & 0.72 & 0.4 & 0.24 \\ -0.96 & -0.96 & 0.768 & -0.768 & 0 & 0 \\ 0.96 & 0.72 & -0.768 & 0.864 & -0.24 & -0.096 \\ 0 & 0.4 & 0 & -0.24 & 0.8 & 0.24 \\ 0 & 0.24 & 0 & -0.096 & 0.24 & 0.096 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$0 = 1.6D_1 + 0.8D_2$$

$$\frac{25.0}{EI} = 0.8D_1 + 2.4D_2$$

$$D_1 = \frac{-6.25}{EI}$$

$$D_2 = \frac{12.5}{EI}$$

$$Q_2 = -0.96EI\left(\frac{-6.25}{EI}\right) - 0.96EI\left(\frac{12.5}{EI}\right) = -6.00 \text{ kN} \quad \text{Ans}$$

$$Q_2 - 30.0 = 0.96EI\left(\frac{-6.25}{EI}\right) + 0.72\left(\frac{12.5}{EI}\right)$$

$$Q_2 = 33 \text{ kN} \quad \text{Ans}$$

$$Q_3 - 25.0 = 0 + 0.4EI\left(\frac{12.5}{EI}\right)$$

$$Q_3 = 30 \text{ kN} \cdot \text{m} \quad \text{Ans}$$

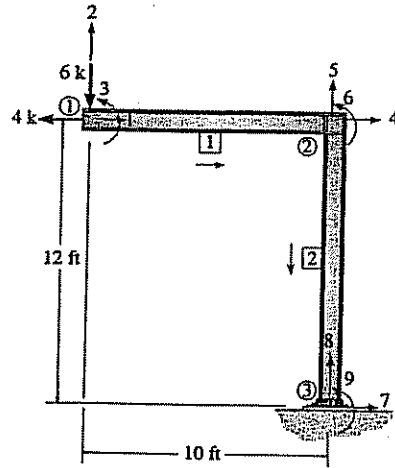
$$Q_4 - 30.0 = 0 + 0.24EI\left(\frac{12.5}{EI}\right)$$

$$Q_4 = 33.0 \text{ kN} \quad \text{Ans}$$

$$\sum \mathcal{M}_1 = 0: 30.0 + 33.0(5) - 60.0(2.5) - 6(7.5) = 0 \quad (\text{Check})$$

$$\sum \mathcal{F}_y = 0: 33.0 + 33.0 - 60.0 - 6 = 0 \quad (\text{Check})$$

*16-4. Determine the components of displacement at ①.
 Take $E = 29(10^3)$ ksi, $I = 650$ in⁴, $A = 20$ in² for each member.



$$D_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad Q = \begin{bmatrix} -4 \\ -6 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 4833.33 & 0 & 0 & -4833.33 & 0 & 0 & 0 & 0 & 0 \\ 0 & 130.90 & 7854.17 & 0 & -130.90 & 7854.17 & 0 & 0 & 0 \\ 0 & 7854.17 & 628333.33 & 0 & -7854.17 & 314166.67 & 0 & 0 & 0 \\ -4833.33 & 0 & 0 & 4909.08 & 0 & 5454.28 & -75.75 & 0 & 5454.28 \\ 0 & -130.90 & -7854.17 & 0 & 4158.68 & -7854.17 & 0 & -4027.78 & 0 \\ 0 & 7854.17 & 314166.67 & 5454.28 & -7854.17 & 1151944.44 & -5454.28 & 0 & 261805.55 \\ 0 & 0 & 0 & -75.75 & 0 & -5454.28 & 75.75 & 0 & -5454.28 \\ 0 & 0 & 0 & 0 & -4027.78 & 0 & 0 & 4027.78 & 0 \\ 0 & 0 & 0 & 5454.28 & 0 & 261805.55 & -5454.28 & 0 & 323411.11 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \\ D_9 \end{bmatrix}$$

Partition matrix

$$\begin{bmatrix} -4 \\ -6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4833.33 & 0 & 0 & -4833.33 & 0 & 0 & 0 & 0 & 0 \\ 0 & 130.90 & 7854.17 & 0 & -130.90 & 7854.17 & 0 & 0 & 0 \\ 0 & 7854.17 & 628333.33 & 0 & -7854.17 & 314166.67 & 0 & 0 & 0 \\ -4833.33 & 0 & 0 & 4909.08 & 0 & 5454.28 & -75.75 & 0 & 5454.28 \\ 0 & -130.90 & -7854.17 & 0 & 4158.68 & -7854.17 & 0 & -4027.78 & 0 \\ 0 & 7854.17 & 314166.67 & 5454.28 & -7854.17 & 1151944.44 & -5454.28 & 0 & 261805.55 \\ 0 & 0 & 0 & -75.75 & 0 & -5454.28 & 75.75 & 0 & -5454.28 \\ 0 & 0 & 0 & 0 & -4027.78 & 0 & 0 & 4027.78 & 0 \\ 0 & 0 & 0 & 5454.28 & 0 & 261805.55 & -5454.28 & 0 & 323411.11 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \\ D_9 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} -4 &= 4833.33D_1 - 4833.33D_4 \\ -6 &= 130.90D_2 + 7854.17D_3 - 130.90D_5 + 7854.17D_6 \\ 0 &= 7854.17D_2 + 628333.33D_3 - 7854.17D_5 + 314166.67D_6 \\ 0 &= -4833.33D_1 + 4909.08D_4 + 5454.28D_7 \\ 0 &= -130.90D_2 - 7854.17D_3 + 4158.68D_5 - 7854.17D_6 \\ 0 &= 7854.17D_2 + 314166.67D_3 + 5454.28D_4 - 7854.17D_5 + 1151944.44D_6 \end{aligned}$$

Solving the above equations yields

$$\begin{aligned} D_1 &= -0.608 \text{ in.} && \text{Ans} \\ D_2 &= -1.12 \text{ in.} && \text{Ans} \\ D_3 &= 0.0100 \text{ rad} && \text{Ans} \\ D_4 &= -0.6076 \text{ in.} \\ D_5 &= -0.001490 \text{ in.} \\ D_6 &= 0.007705 \text{ rad} \end{aligned}$$