# TWOD

## A Finite Element Computer Program for Two-Dimensional Elastostatic Analyses

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## I. INTRODUCTION

The computer program TWOD described herein was developed to perform finite element analyses of two-dimensional elastic continuua under static conditions. TWOD is intended for instructional use. It is thus liberally commented and generally uses descriptive variable names.

## II. UNITS

The units used for various input quantities must be consistent, and determine the units of the output. In the subsequent description of the program input, the units associated with input quantities are described using the following conventions:

F : denotes units of force (e.g., newtons, dynes, pounds, etc.)
L : denotes units of length (e.g., meters, centimeters, inches, etc.)
t : denotes units of time (e.g., seconds, minutes, etc.)

# III. PROGRAM INPUT

Finite element analyses generally require large amounts of input data. The preparation of this data should be as easy as possible and should leave little room for error. In keeping with these concerns, the input data associated with the TWOD computer program contains simple keywords and numeric values grouped into "blocks".

The beginning of each data block is indicated by the occurrence of one of seven "block" keywords:

TITLE
GENERAL
MATERIALS
NODES
ELEMENTS
CONCENTRATED SPECIFICATIONS
DISTRIBUTED SPECIFICATIONS

Following a keyword, alphanumeric data is specified within each data block. Finally, each block is terminated with the END keyword. For example, the block used in specifying material properties would be entered as

```
MATERIALS
     1     2     0.0    150.0
     30.0E+06     0.25
END
```

All keywords associated with TWOD:

- must be entered beginning in the *first* column of a new line;
- may be specified using *entirely* uppercase letters or *entirely* lowercase letters; and,
- may be abbreviated by specifying at least their first *three* letters.

The material data given in the previous example could thus also be entered as:

```
materials
     1     2     0.0    150.0
     30.0E+06     0.25
end
```

or as

```
MAT
     1     2     0.0    150.0
     30.0E+06     0.25
```

```
END
```

In order to improve the readability of input data files, two comment characters – the asterisk "*" and the sharp "!" are provided. If an asterisk or sharp is entered into the first column of a data line, the remainder of the line will be ignored by the program and the next line will automatically be read. Comment lines thus represent a convenient way to annotate input data files. For example, the above material data material might be entered as

```
material
*
*         parameter values for steel
*
     1     2    0.0   150.0
     30.0E+06     0.25
!
!    parameter values for 304 STAINLESS STEEL
!
     2     1    0.0   105.0
     30.0E+06     0.25
end
```

All numeric input to TWOD is performed using list-directed READ statements (i.e., free-format). Since input is terminated by exhaustion of the input list rather than by line boundaries, list-directed input records may span several lines. Multiple spaces between input fields are ignored and "null" fields in the input record may leave the value of a particular variable unchanged (as opposed to setting it to zero). *The safest way to avoid erroneous input is to include explicit values for all input variables that are being read*; this ensures that the correct mathematical model will be analyzed. Users not familiar with list-directed input are advised to consult appropriatedocumentation for their computer. When alphanumeric strings and/or keywords are required in the input, each of them should appear on a separate line and *must not be preceded by a blank record.*

In the subsequent description of the input quantities required by the program, the variable types (i.e., alphanumeric, integer, or real) are given in brackets [ ]. Numbers in braces { } signify that a note explaining the particular input quantity is provided in the following section. Since the safest way to avoid erroneous list-directed input is to include explicit values for all variables that are being read, no default values are set within the program. *Note also that only very rudimentary error checking is performed within the TWOD computer program.*

The data required by the program consists of the following quantities:

## III.1 Information Describing the Analysis

- Supply a line with the keyword **TITLE** entered, beginning in the first column.

- Then supply as many lines of information as necessary to describe the analysis to be performed. The maximum length of each line is 80 characters.

- To complete the block of descriptive information, supply a line with the keyword **END** entered, beginning in the first column.

♦ EXAMPLE:

```
TITLE
   This example shows how to supply alphanumeric information
   describing a particular analysis. Any information supplied
   in this “block” will be printed at the beginning of the
   output file.
END
```

## III.2 General Analysis Options

• Supply a line with the keyword **GENERAL** entered, beginning in the first column.

• Then supply the following information – in any order but beginning in column one – about the analysis to be performed.

**PLANE STRESS** : perform a plane stress analysis

or

**PLANE STRAIN** : perform a plane strain analysis

**CHECK** : perform only a check of the input data

or

**ANALYZE** : perform a complete analysis

• To complete the general analysis option specifications, supply a line with the keyword **END** entered, beginning in the first column.

♦ EXAMPLE:

A check of the mathematical model to be used in a plane stress analysis is first desired. The associated general analysis options data block is thus

```
general
check
plane stress
end
```

Once the correctness of the input data is verified and the analysis is to be performed, the above data is modified in the following manner:

```
general
analyze
plane stress
end
```

## III.3 Material Property Specifications {1}

- Supply a line with the keyword **MATERIALS** entered, beginning in the first column.

- The following information must be specified *for each material type*.

**NUM** [integer] = material number

**MATYPE** [integer] = 1 : isotropic linear elastic material idealization
or
2 : anisotropic linear elastic material idealization

**BFORCX** [real] = value of the body force (units : $FL^{-3}$) associated with the x-direction [1].

**BFORCY** [real] = value of the body force (units : $FL^{-3}$) associated with the y-direction [1].

- Beginning on a new line, supply the following information:

–> if **MATYPE** = 1 :

**EMOD** [real] = value of the elastic modulus[2] (units : $FL^{-2}$).

**PRATIO** [real] = value of Poisson's ratio[2].

–> else if **MATYPE** = 2 : {2}

**ANGLE** [real] = angle between global $x_1$-axis and material $x_1$-axis (degrees, measured counterclockwise).

**C11** [real] = value of an anisotropic material parameter [3] (units : $FL^{-2}$).

**C12** [real] = value of an anisotropic material parameter (units : $FL^{-2}$).

**C13** [real] = value of an anisotropic material parameter (units : $FL^{-2}$).

---

1 For a given element, the body force components are assumed *constant*.

2 Over a given element, the elastic modulus and Poisson's ratio are assumed *constant*.

3 Over a given element, the anisotropic material parameters are assumed *constant*.

## III.3 Material Property Specifications (continued)

**C14** [real] = value of an anisotropic material parameter (units : $FL^{-2}$).

**C22** [real] = value of an anisotropic material parameter (units : $FL^{-2}$).

**C23** [real] = value of an anisotropic material parameter (units : $FL^{-2}$).

**C24** [real] = value of an anisotropic material parameter (units : $FL^{-2}$).

**C33** [real] = value of an anisotropic material parameter (units : $FL^{-2}$).

**C34** [real] = value of an anisotropic material parameter (units : $FL^{-2}$).

**C44** [real] = value of an anisotropic material parameter (units : $FL^{-2}$).

*NOTE*: in specifying values for the parameters C11 – C44, more than one line may be used.

• To complete the material property data block, supply a line with the keyword **END** entered, beginning in the first column.

♦ EXAMPLE:

In a particular analysis, material number 1 is isotropic linear elastic; material number 2 is anisotropic. The following material property data block is thus associated with the analysis:

```
MATERIAL
   1  1    0.0   0.10
   30.0e+06  0.3
*
   2  2    0.0  0.10
   4.038e07 1.731e07 1.731e07 0.0
            4.038e07 1.731e07 0.0
                     4.038e07 0.0
                              1.154e07
END
```

## III.4 Nodal Coordinate Data {3}

• Supply a line with the keyword **NODES** entered, beginning in the first column.

• The following information is repeated as many times as necessary to specify the locations of all nodes in the model which are not to be generated by means of the "interior" generation scheme (see the section titled "Explanatory Notes Regarding Program Input").

| | | |
|---|---|---|
| **NUM** | [integer] | = node point number |
| **XP** | [real] | = x coordinate of the node point (units : L ) |
| **YP** | [real] | = y coordinate of the node point (units : L ) |
| **INCR** | [integer] | = numbering increment for successive node points [4]. |
| **RATIO** | [real] | = spacing ratio for successive node points [4]. |
| **NUMADD** | [integer] | = number of additional points used in the interpolation of nodal coordinates. If **NUMADD** = 0, *linear* interpolation is used. If **NUMADD** = 1, *quadratic* interpolation is used [4]. |

–> if **NUMADD** = 1 : input, on the same line as the above data, the following quantities:

| | | |
|---|---|---|
| **XT** | [real] | = x coordinate of an intermediate point through which the nodal interpolation will pass (units : L ). |
| **YT** | [real] | = y coordinate of an intermediate point through which the nodal interpolation will pass (units : L ). |

• To complete the nodal coordinate data block, supply a line with the keyword **END** entered, beginning in the first column.

[4] This quantity is associated with the linear or quadratic generation options; further details are given in program note {3}.

## III.5 Element Data {5}

The following information is required to define the topology of all elements in the model being analyzed. The order of the element records need bear no relation to the actual locations of the elements within the body. The order will, however, determine the assigned element numbers. In the current version of TWOD, the following elements are available:

2-node bar (axial) elements (see Figure 1);
3-node triangular elements (see Figure 2);
4-node quadrilateral elements (see Figure 3);
6-node triangular elements (see Figure 4);
8-node (“serendipity”) quadrilateral elements (see Figure 5);
9-node (Lagrange) quadrilateral elements (see Figure 6); and,
4-node quadrilateral non-conforming elements (see Figure 3).

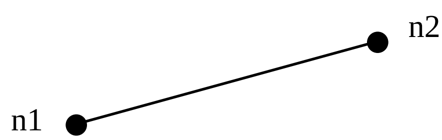



**Figure 1**
Typical 2-Node Bar Element

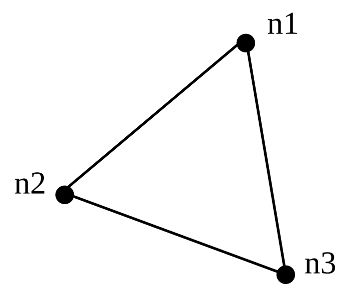



**Figure 2**
Typical 3-Node Triangular Element

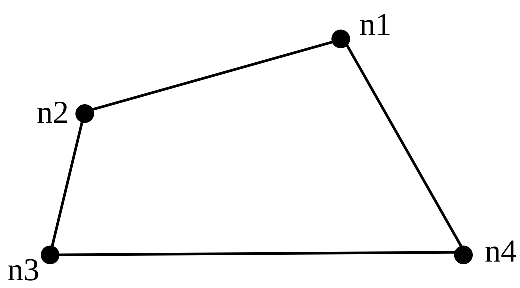



**Figure 3**
Typical 4-Node Quadrilateral Element

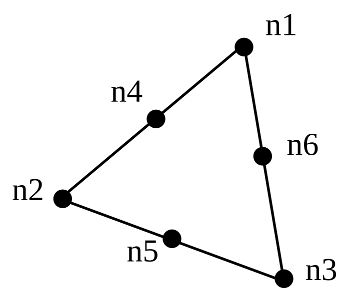



**Figure 4**
Typical 6-Node Triangular Element

## III.5 Element Data (continued)

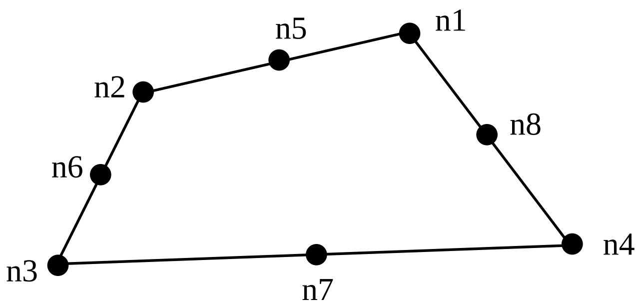



**Figure 5**
Typical 8-Node Quadrilateral Element

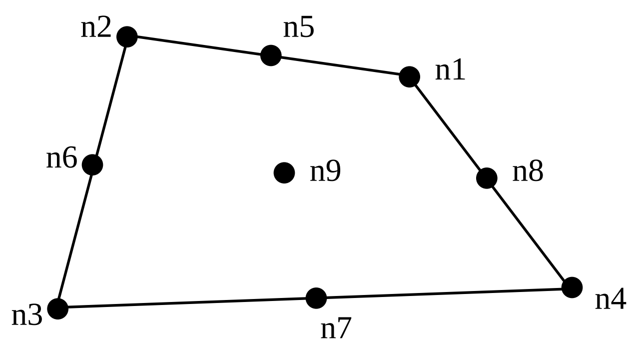



**Figure 6**
Typical 9-Node Quadrilateral Element

- To begin the description of element data, supply a line with the keyword **ELEMENTS** entered, beginning in the first column.

## III.5 Element Data (continued)

–> if two-node bar (axial) elements are desired, supply the following information:

- Supply a line with the keyword **BAR** entered, beginning in the first column.

- Next for *all* two-node bar elements found in the mesh, supply the following information (given on a separate line).

**N1**, **N2** } = node numbers which describe the element (see Figure 1)

**MNUM** [integer] = number of the material associated with the element. {4}
**AREA** [real] = cross-sectional are of the element [5].
**NUMADD** [integer] = number of additional elements in the layer.
If element generation is *not* desired, set **NUMADD** = 0.
**INCADD** [integer] = numbering increment for nodes in the layer.

- To terminate the description of two-node bar elements, supply a line with the keyword **END** entered, beginning in the first column.

♦ EXAMPLE:

The following data is used to describe the line of bar (axial) elements shown in the figure below.

```
ELEMENTS
bar
    1   2   1   0.10  4  1
end
END
```

[5] Over a given element, the cross-sectional area is assumed *constant*.

## III.5 Element Data (continued)

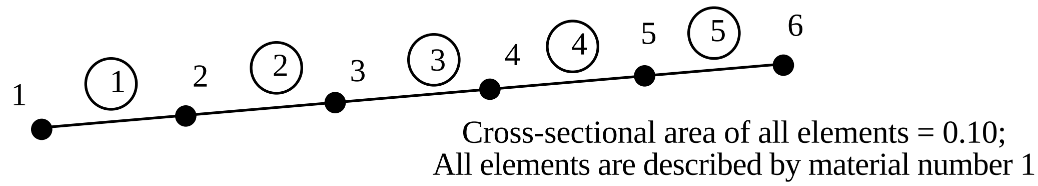



**Figure 7**

Line of Bar Elements

## III.5 Element Data (continued)

–> if three-node triangular elements are desired, supply the following information:

- Supply a line with the keyword **3-NODE TRIANGLES** entered, beginning in the first column.

- Next for *all* three-node triangles found in the mesh, supply the following information (given on a separate line).

**N1** [integer]
**N2** [integer] } = node numbers which describe the element [6] (see Figure 2)
**N3** [integer]

**MNUM** [integer] = number of the material associated with the element. {4}

**ETHICK** [real] = element thickness (for plane strain idealizations, set **ETHICK** = 1.0)

- To terminate the description of three-node triangular elements, supply a line with the keyword **END** entered, beginning in the first column.

---

[6] The node numbers must be input *counterclockwise* around the element (see Figure 2 above). No generation options currently exist for triangular elements.

## III.5 Element Data (continued)

–> if four-node quadrilateral elements are desired, supply the following information:

• Supply a line with the keyword **4-NODE QUADS** entered, beginning in the first column.

• Next for *all* four-node quadrilaterals found in the mesh, supply the following information (given on a separate line).

**N1** [integer]
**N2** [integer]
**N3** [integer]
**N4** [integer]
} = node numbers which describe the element [7] (see Figure 3)

**MNUM** [integer] = number of the material associated with the element. {4}

**ETHICK** [real] = element thickness (for plane strain idealizations, set **ETHICK** = 1.0)

**NUMADD** [integer] = number of additional elements in the layer.
If element generation is *not* desired, set **NUMADD** = 0.

**INCADD** [integer] = numbering increment for nodes in the layer.

**NUMLAY** [integer] = number of additional layers.
If element generation is *not* desired, set **NUMLAY** = 0.

**INCLAY** [integer] = numbering increment between nodes in the additional layers.

• To terminate the description of four-node quadrilateral elements, supply a line with the keyword **END** entered, beginning in the first column.

[7] The node numbers must be input *counterclockwise* around the element (see Figure 3 above).

## III.5 Element Data (continued)

♦ EXAMPLE:

The following data is used to describe the grid of four-node quadrilateral elements shown in the figure below. All elements are assumed to be described by material number 1, and to have a thickness equal to 1.20 (plane stress analysis).

```
ELEMENTS
4-node quads
    1  6  7  2  1  1.20   4  5  3  1
end
END
```

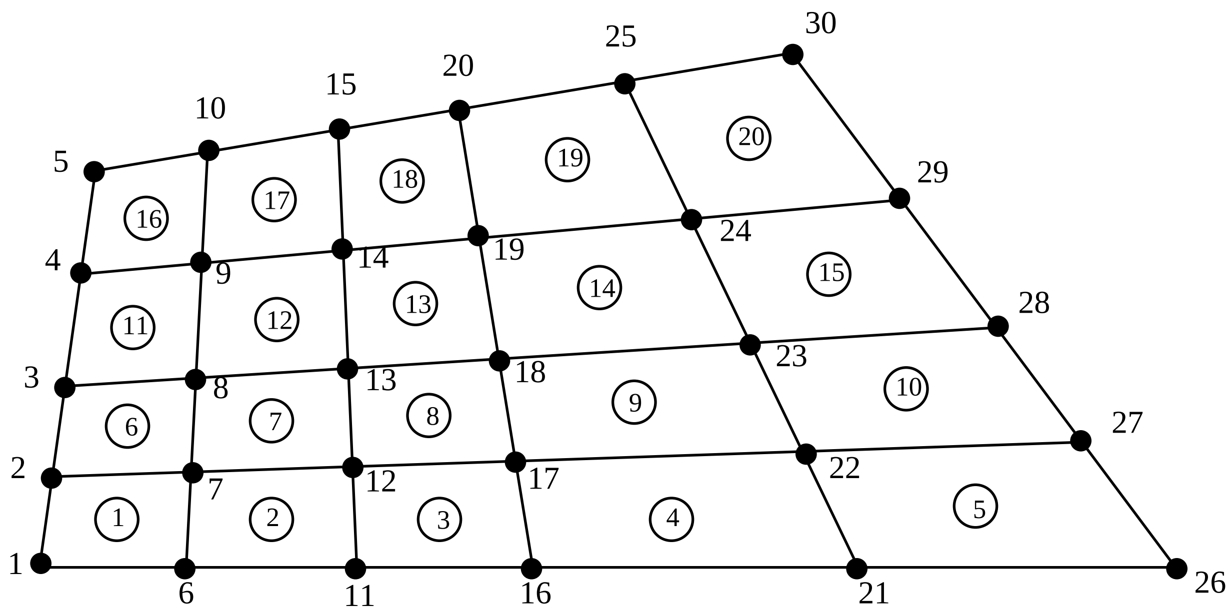



**Figure 8**

Sample Finite Element Mesh

Consisting of 4-Node Quadrilateral Elements

## III.5 Element Data (continued)

–> if six-node triangular elements are desired, supply the following information:

- Supply a line with the keyword **6-NODE TRIANGLES** entered, beginning in the first column.

- Next for *all* six-node triangles found in the mesh, supply the following information (given on a separate line).

**N1** [integer]
**N2** [integer]
**N3** [integer]
**N4** [integer]
**N5** [integer]
**N6** [integer]
} = node numbers which describe the element [8] (see Figure 4)

**MNUM** [integer] = number of the material associated with the element. {4}

**ETHICK** [real] = element thickness (for plane strain idealizations, set **ETHICK** = 1.0)

- To terminate the description of six-node triangular elements, supply a line with the keyword **END** entered, beginning in the first column.

---

[8] The node numbers must be input *counterclockwise* around the element (see Figure 4 above). No generation options currently exist for triangular elements.

## III.5 Element Data (continued)

–> <u>if eight-node quadrilateral elements are desired, supply the following information</u>:

- Supply a line with the keyword **8-NODE QUADS** entered, beginning in the first column.

- Next for *all* eight-node quadrilaterals found in the mesh, supply the following information (given on a separate line).

| | | |
|---|---|---|
| **N**1 | [integer] | |
| **N**2 | [integer] | |
| **N**3 | [integer] | |
| **N**4 | [integer] | = node numbers which describe the element [9](see Figure 5) |
| **N**5 | [integer] | |
| **N**6 | [integer] | |
| **N**7 | [integer] | |
| **N**8 | [integer] | |

| | | | |
|---|---|---|---|
| **MNUM** | [integer] | = number of the material associated with the element. | {4} |
| **ETHICK** | [real] | = element thickness (for plane strain idealizations, set **ETHICK** = 1.0) | |
| **NUMADD** | [integer] | = number of additional elements in the layer. If element generation is *not* desired, set **NUMADD** = 0. | |
| **INCADD** | [integer] | = numbering increment for nodes in the layer. | |
| **NUMLAY** | [integer] | = number of additional layers. If element generation is *not* desired, set **NUMLAY** = 0. | |
| **INCLAY** | [integer] | = numbering increment between nodes in the additional layers. | |

- To terminate the description of eight-node quadrilateral elements, supply a line with the keyword **END** entered, beginning in the first column.

[9] The node numbers must be input *counterclockwise* around the element (see Figure 5 above).

## III.5 Element Data (continued)

–> <u>if nine-node quadrilateral elements are desired, supply the following information</u>:

- Supply a line with the keyword **9-NODE QUADS** entered, beginning in the first column.

- Next for *all* nine-node quadrilaterals found in the mesh, supply the following information (given on a separate line).

| | | |
|---|---|---|
| **N1** | [integer] | |
| **N2** | [integer] | |
| **N3** | [integer] | |
| **N4** | [integer] | |
| **N5** | [integer] | = node numbers which describe the element [10] (see Figure 6) |
| **N6** | [integer] | |
| **N7** | [integer] | |
| **N8** | [integer] | |
| **N9** | [integer] | |

| | | |
|---|---|---|
| **MNUM** | [integer] | = number of the material associated with the element. {4} |
| **ETHICK** | [real] | = element thickness (for plane strain idealizations, set **ETHICK** = 1.0) |
| **NUMADD** | [integer] | = number of additional elements in the layer. If element generation is *not* desired, set **NUMADD** = 0. |
| **INCADD** | [integer] | = numbering increment for nodes in the layer. |
| **NUMLAY** | [integer] | = number of additional layers. If element generation is *not* desired, set **NUMLAY** = 0. |
| **INCLAY** | [integer] | = numbering increment between nodes in the additional layers. |

- To terminate the description of nine-node quadrilateral elements, supply a line with the keyword **END** entered, beginning in the first column.

[10] The node numbers must be input *counterclockwise* around the element (see Figure 6 above).

## III.5 Element Data (continued)

–> <u>if four-node non-conforming quadrilateral elements are desired, supply the following information</u>:

• Supply a line with the keyword **QM6** entered, beginning in the first column.

• Next for *all* four-node quadrilaterals found in the mesh, supply the following information (given on a separate line).

| | | |
|---|---|---|
| **N1** | [integer] | = node numbers which describe the element [11] (see Figure 3) |
| **N2** | [integer] | |
| **N3** | [integer] | |
| **N4** | [integer] | |

| | | | |
|---|---|---|---|
| **MNUM** | [integer] | = number of the material associated with the element. | {4} |
| **ETHICK** | [real] | = element thickness (for plane strain idealizations, set **ETHICK** = 1.0) | |
| **NUMADD** | [integer] | = number of additional elements in the layer. If element generation is *not* desired, set **NUMADD** = 0. | |
| **INCADD** | [integer] | = numbering increment for nodes in the layer. | |
| **NUMLAY** | [integer] | = number of additional layers. If element generation is *not* desired, set **NUMLAY** = 0. | |
| **INCLAY** | [integer] | = numbering increment between nodes in the additional layers. | |

• To terminate the description of four-node quadrilateral elements, supply a line with the keyword **END** entered, beginning in the first column.

• To complete the element data block, supply a line with the keyword **END** entered, beginning in the first column.

[11] The node numbers must be input *counterclockwise* around the element (see Figure 3 above).

## III.6 Distributed Nodal Specifications {6}

The following information is required to specify all distributed tractions applied to the mathematical model. TWOD automatically converts these tractions into suitable *equivalent* nodal forces.

• Supply a line with the keyword **DISTRIBUTED** entered, beginning in the first column.

–> For all distributed nodal specifications, applied to three-node triangles or to four-node quadrilateral elements, supply the following information: {7}

• Supply a line with the keywords **3-NODE TRIANGLES** or **4-NODE QUADS** entered, beginning in the first column (*NOTE*: for QM6 elements, enter the keyword **4-NODE QUADS**)

• Next for elements on which distributed tractions are applied, supply the following information (beginning on a separate line).

**NUM** [integer] = number of node point at which the specification is to be applied.

**NUMEND** [integer] = number of final node point in the sequence being generated. {8}

**INCR** [integer] = numbering increment between successive nodes in the sequence. If data generation is *not* used, set **INCR** equal to zero. {8}

**PN(1)** [real] = value of linearly varying normal traction at *first* node in the sequence (units: $FL^{-1}$). {9}

**PT(1)** [real] = value of linearly varying tangential traction at *first* node in the sequence (units: $FL^{-1}$). {9}

**PN(2)** [real] = value of linearly varying normal traction at *last* node in the sequence (units: $FL^{-1}$). {9}

**PT(2)** [real] = value of linearly varying tangential traction at *last* node in the sequence (units: $FL^{-1}$). {9}

• To terminate the description of distributed nodal specifications for three-node triangles or for four-node quadrilateral elements, supply a line with the keyword **END** entered, beginning in the first column.

## III.6 Distributed Nodal Specifications (continued)

–> For all distributed nodal specifications, applied to six-node triangular elements and/or to eight- and/or nine-node quadrilateral elements, supply the following information: {10}

- Supply a line with the keywords **6-NODE TRIS**, **8-NODE QUADS** or **9-NODE QUADS** entered, beginning in the first column.

- Next for elements on which distributed tractions are applied, supply the following information (beginning on a separate line).

**NUM** [integer] = number of node point at which the specification is to be applied.

**NUMEND** [integer] = number of final node point in the sequence being generated.{11}

**INCR1** [integer] = numbering increment between successive *corner* nodes in the sequence. If data generation is *not* used, set **INCR1** equal to zero. {11}

**INCR2** [integer] = numbering increment between *corner and adjacent mid-side* nodes in the sequence. If data generation is *not* used, set **INCR2** equal to zero.

**PN(1)** [real] = value of normal traction at *first* node in the sequence (units: $FL^{-1}$). {9}

**PT(1)** [real] = value of tangential traction at *first* node in the sequence (units: $FL^{-1}$). {9}

**PN(3)** [real] = value of linearly varying normal traction at *last* node in the sequence (units: $FL^{-1}$). {9}

**PT(3)** [real] = value of linearly varying tangential traction at *last* node in the sequence (units: $FL^{-1}$). {9}

**NEXTRA** [integer] = { 0 : linear interpolation used in traction specification; 1 : quadratic interpolation used in traction specification } {12}

## III.6 Distributed Nodal Specifications (continued)

**XT** [real] = x-coordinate of an intermediate point at which the traction is specified. A value for **XT** is input, on the *same line* as the above data, only if **NEXTRA** = 1.

**YT** [real] = y-coordinate of an intermediate point at which the traction is specified. A value for **YT** is input, on the *same line* as the above data, only if **NEXTRA** = 1.

**PN(2)** [real] = value of linearly varying normal traction at the *intermediate* node in the sequence (units: $FL^{-1}$).

**PT(2)** [real] = value of linearly varying tangential traction at the *intermediate* node in the sequence (units: $FL^{-1}$).

- To terminate the description of distributed nodal specifications for six-, eight- and/or nine-node elements, supply a line with the keyword **END** entered, beginning in the first column.

- To complete the distributed nodal specification data block, supply a line with the keyword **END** entered, beginning in the first column.

## III.7 Concentrated Nodal Specifications {13}

The following information is required to specify all concentrated nodal quantities associated with a given mathematical model.

- Supply a line with the keyword **CONCENTRATED** entered, beginning in the first column.

- The following information is repeated as many times as necessary to describe all such specified nodal information.

**NUM** [integer] = number of node point at which the specification is to be applied.

**NUMEND** [integer] = number of final node point in the sequence being generated. {14}

**INCR** [integer] = numbering increment between nodes in the sequence. {14}

**THETA** [real] = angle (in degrees) between the local $x_1$-axis and the global x-axis (measured *counterclockwise* from the global x-axis). {15}

**IFLAG1** [integer] = {16}
- 0 : known *force* is specified in the $x_1$-direction
- 1 : known *displacement* is specified in the $x_1$-direction
- 2 : *linear spring* is specified in the $x_1$-direction

**VALUE1** [real] = magnitude of specified { force (units : F); displacement (units : L); spring stiffness (units : $FL^{-1}$) } in $x_1$-direction

## III.7 Concentrated Nodal Specifications (continued)

**IFLAG**2 [integer] = { 0 : known *force* is specified in the $x_2$-direction; 1 : known *displacement* is specified in the $x_2$-direction; 2 : *linear spring* is specified in the $x_2$-direction } {17}

**VALUE**2 [real] = magnitude of specified { force (units : F); displacement (units : L); spring stiffness (units : $FL^{-1}$) } in $x_2$-direction

• To complete the concentrated nodal specification data block, supply a line with the keyword **END** entered, beginning in the first column.

## III.8 Data Input Termination Record

• To signify the termination of the input data, supply a line with the keyword **FINISHED** entered, beginning in the first column.

# IV. EXPLANATORY NOTES REGARDING PROGRAM INPUT

{1} The units associated with the material property specifications must be consistent with those used to describe the geometry of the body being analyzed and with the applied displacements and/or forces. The material number **NUM** serves as an identifier in the assignment of particular material descriptions to groups of elements in Section 5 of the input.

{2} For an anisotropic linear elastic material idealization, the constitutive relation is written in the form:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{12} & C_{22} & C_{23} & C_{24} \\ C_{13} & C_{23} & C_{33} & C_{34} \\ C_{14} & C_{24} & C_{34} & C_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \end{Bmatrix}$$

where $\sigma_x$ , $\sigma_y$ and $\sigma_z$ represent normal stress components in the x-, y- and z-directions; $\tau_{xy}$ represents the shear stress; $\varepsilon_x$ , $\varepsilon_y$ and $\varepsilon_z$ represent components of normal strain; and, $\gamma_{xy}$ represents the "engineering" measure of shear strain.

{3} The coordinates of each node point must be defined, but need not be input in any particular order. To assist the analyst in defining these coordinates, the TWOD computer program offers *three* data generation schemes. Nodal coordinates can be generated along a straight line (this is achieved using linear interpolation), along a curve (this is achieved using quadratic interpolation), or within part or all of the interior of the mesh. The use of these schemes can, for example, enable one to describe the nodal coordinates of an arbitrarily large grid with as few as five input records.

When numbering the nodes, not all numbers between 1 and the highest node number (NUMNOD) need correspond to actual nodes in the body. For example, the numbering scheme shown in Figure 9 is perfectly permissible (with no decrease in program efficiency); coordinates for the non-existent nodes 15 and 21 need not be specified. This feature facilitates the use of the node point and element generation options (to be subsequently discussed).

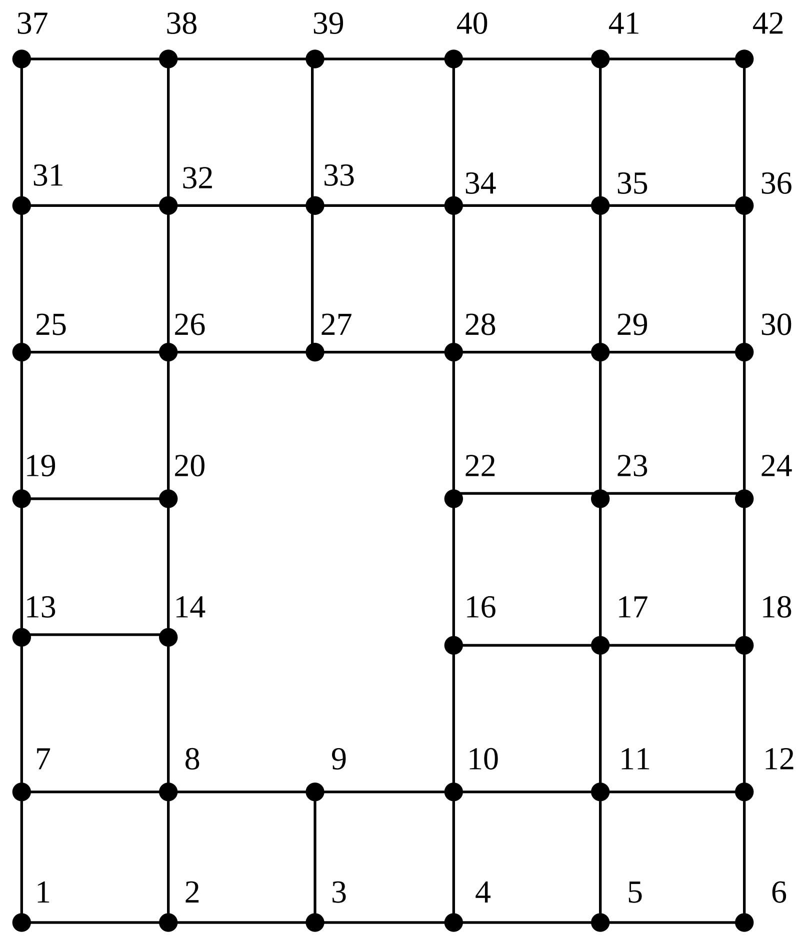



**Figure 9**
Sample Mesh with Missing Node Numbers

If the location of a node is prescribed more than once in the input and the coordinates are not in agreement, the last description is used. If, however, in a second (or subsequent description) the node number is entered as *negative*, then the previous location associated with this node is assumed. The utility of this option will subsequently be illustrated. Whenever several sequential node points lie along a straight line or along a curve, the coordinate generation option, with either linear or quadratic interpolation may be used. If such a situation exists, it is only necessary to enter the coordinates of the initial and final points of the sequence (denoted in this discussion by N and N´, respectively), and the values of **INCR**, **RATIO**, **NUMADD** and possibly **XT** and **YT**. The constant **INCR** represents the difference between any two successive node numbers in the sequence and **RATIO** defines the ratio of the distances between any two adjacent pairs of node points. The value selected for **NUMADD** determines whether linear or quadratic interpolation will be used.

–> *Linear Interpolation*:

This generation option is realized by setting **NUMADD** equal to zero (0); in this case no input is required for the quantities **XT** and **YT** . If, for the input record describing node N´, **INCR** ≠ 0, then intermediate node points will be generated along a straight line between node N´ and the point N described in the *preceding* node specification record. That is, the coordinates of the points (N + **INCR**), (N + 2***INCR**, . . . , N´ – **INCR**) will each be automatically generated (see Figure 10).

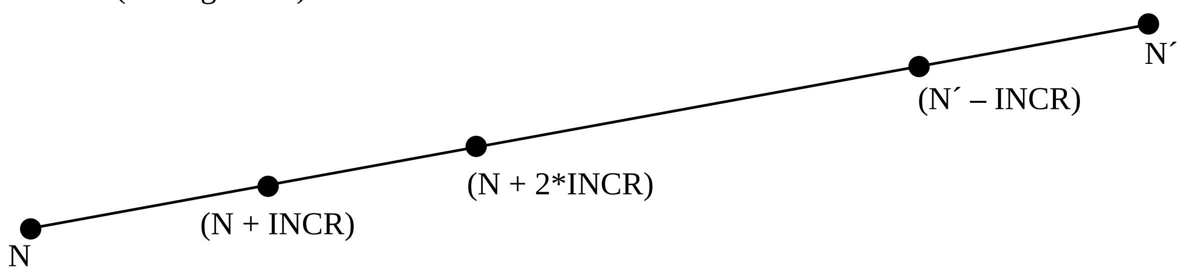



**Figure 10**
Nodal Generation Using Linear Interpolation

The node number N´, for which the specified non-zero value of **INCR** triggered the generation of the line N –> N´, can also serve as the initial point of a line generated between it and the point described in the *following* record. The *exterior* nodes shown in Figure 9 can thus be generated with the following five records (the quantities $x_i$ and $y_i$ represent the x- and y-coordinates of node "i" ):

```
NODES
  1    x1   y1    0   1.0    0
  6    x6   y6    1   1.0    0
 42    x42  y42   6   1.0    0
 37    x37  y37  -1   1.0    0
 -1     x1   y1  -6   1.0    0
END
```

The end points of a line sequence may be entered in either order. For example, the segments illustrated in Figure 11 can be defined by specifying the nodes in *either* the order 7 –> 22; i.e.,

```
  7     x7    y7     0   1.0    0
 22    x22   y22     5   2.0    0
```

or in the order 22 –> 7; i.e.,

```
22    x22   y22    0   1.0    0
 7     x7    y7   -5   0.5    0
```

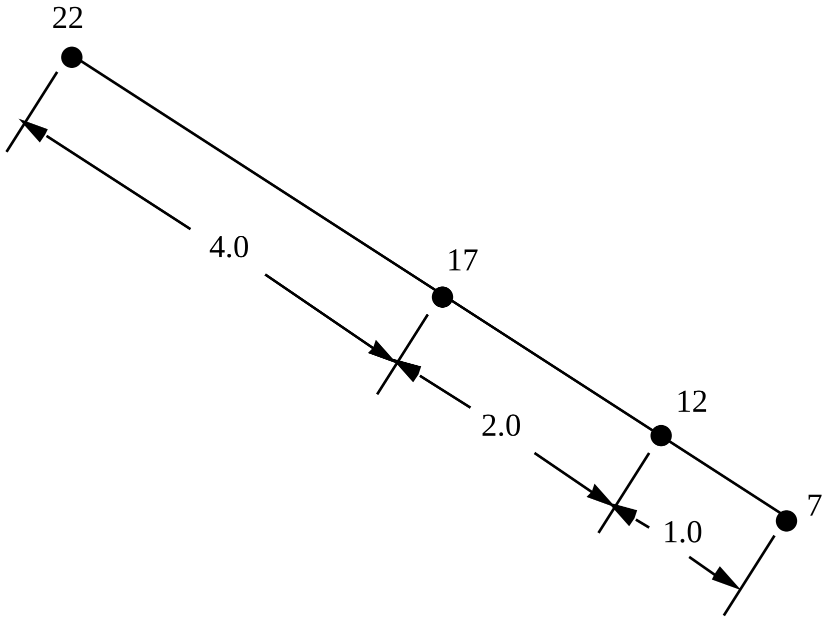



**Figure 11**
Example of Linear Nodal Coordinate Generation

The spacing of the intermediate points (e.g., nodes 12 and 17 in the above example) is controlled by the spacing ratio **RATIO**. A value of **RATIO** = 1.0 results in equally spaced nodes. In selecting a value of **RATIO**, the following three formulas are of use (in the sequel the number of segments comprising a sequence of nodes is denoted by “m”):

1) If the total length of a line of nodes is equal to L and if **RATIO** ≠ 1.0, then the length of the first segment will equal

$$\frac{L\,(1 - \mathbf{RATIO})}{(1 - \mathbf{RATIO}^{m})}$$

2) If the ratio of the lengths of the last and the first segments must equal a specified value “D” (i.e., $D = \Delta L_m/\Delta L_1$ ), then specify a value of **RATIO** equal to $D^r$, where $r = (m - 1)^{-1}$ and where $m > 1$.

3) Suppose a line of nodes has been generated with a particular value of **RATIO** and that this line is to now be regenerated with *twice* as many segments (i.e., with twice as many node points minus one). Furthermore suppose that it is desired to have the locations of the first set of nodes be retained in the second set. To achieve the above-mentioned goals use $\sqrt{\textbf{RATIO}}$ as the new spacing ratio.

–> *Quadratic Interpolation*:

In this option quadratic isoparametric generation is employed, and facilitates the placement of node points along curved lines. The option is realized by setting **NUMADD** equal to one (1). The quantities **XT** and **YT** represent the x- and y-coordinates of an intermediate point through which the line of nodes will pass. In general, this intermediate point does not coincide with any node point.

For the case of quadratic interpolation, the location of a node point is determined from the following mapping (see also Figure 12):

$$x_p = x(\xi_p) = \frac{1}{2}\xi_p(\xi_p - 1)\,x_N + (1 - \xi_p^2)\,XT + \frac{1}{2}\xi_p(\xi_p + 1)\,x_{N'}$$

$$y_p = y(\eta_p) = \frac{1}{2}\eta_p(\eta_p - 1)\,y_N + (1 - \eta_p^2)\,YT + \frac{1}{2}\eta_p(\eta_p + 1)\,y_{N'}$$

where

$(x_p, y_p)$ = the coordinates, in (x, y) space, of node point “p”;

$(x_N, y_N)$ = the coordinates, in (x, y) space, of the initial node in the sequence;

$(x_{N'}, y_{N'})$ = the coordinates, in (x, y) space, of the final node in the sequence;

$\xi, \eta$ = local (natural) coordinates ( $-1 \le \xi, \eta \le 1$) defined by

$$\xi \equiv \frac{x - 0.5\,(x_{N'} + x_N)}{x_{N'} - x_N}\ , \quad \eta \equiv \frac{y - 0.5\,(y_{N'} + y_N)}{y_{N'} - y_N}\ ; \text{ and,}$$

$(\xi_p, \eta_p)$ = coordinates of node "p" in $(\xi, \eta)$ space.

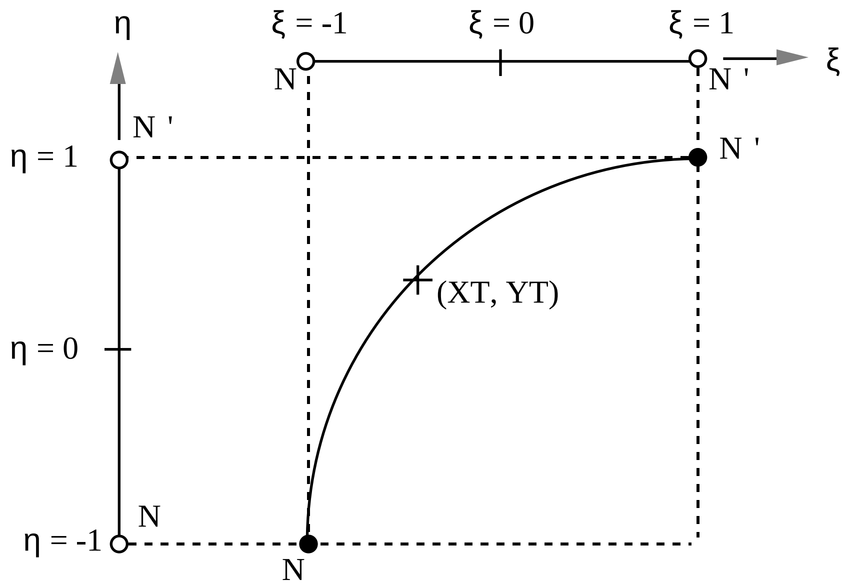



**Figure 12**
Nodal Coordinate Generation Using Quadratic Interpolation

The description of the quantities **INCR** and **RATIO** given in conjunction with the description of linear interpolation applies directly to the quadratic interpolation, only in $(\xi, \eta)$ space.

Without any prompting of the user the interior node point generation option automatically locates all interior nodes whose coordinates have not been established through the options cited above (that is, all node points left undefined after the input of Section 3 has been processed; note that all boundary nodes must be directly or indirectly specified by the input to Section 3). The locations of the undefined interior nodes are computed by means of the "Isoparametric" grid generation scheme described by Herrmann[12] and subsequently generalized by the author [13].

[12] L. R. Herrmann, "Laplacian – Isoparametric Grid Generation Scheme," *Journal of the Engineering Mechanics Division, ASCE*, Vol. 102, No. 5, October 1976, pp. 749-756.

[13] V. N. Kaliakin, "A Simple Coordinate Determination Scheme for Two-Dimensional Mesh Generation", *Computers and Structures*, Vol. 43, No. 3, May 1992, pp. 505-516.

Two grids prepared with the aid of the grid generation schemes are shown in Figure 13. Grid #1 was developed using the straight line generation scheme to specify the exterior (boundary) nodes, and the interior generation scheme to determine the coordinates of the remaining nodes. Only the following records are required in Section 3 to construct the mesh (the nodes along each line are equally spaced):

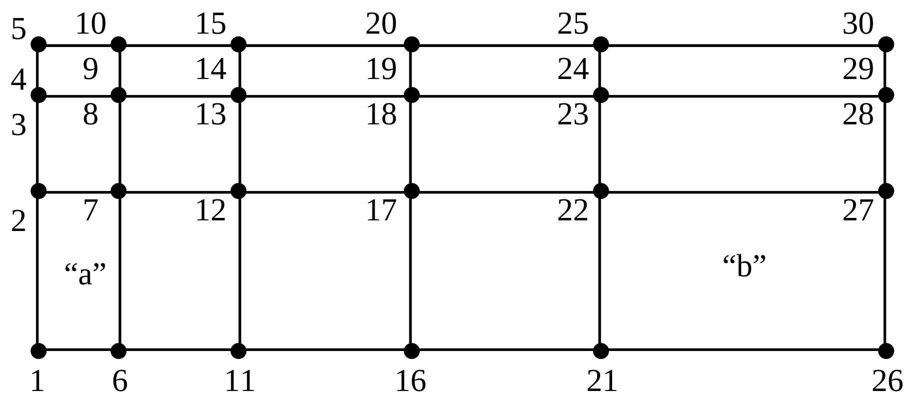
| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | $x_1$ | $y_1$ | 0 | 1.0 | 0.0 | 0.0 |
| 26 | $x_{26}$ | $y_{26}$ | 5 | 1.25 | 0.0 | 0.0 |
| 30 | $x_{30}$ | $y_{30}$ | 1 | 0.50 | 0.0 | 0.0 |
| 5 | $x_5$ | $y_5$ | -5 | 0.80 | 0.0 | 0.0 |
| -1 | 0.0 | 0.0 | -1 | 2.0 | 0.0 | 0.0 |

Since the specification of node 1 for a second time employs the negative of the node number, the nodal coordinates, which are read and ignored, are simply input as zero.

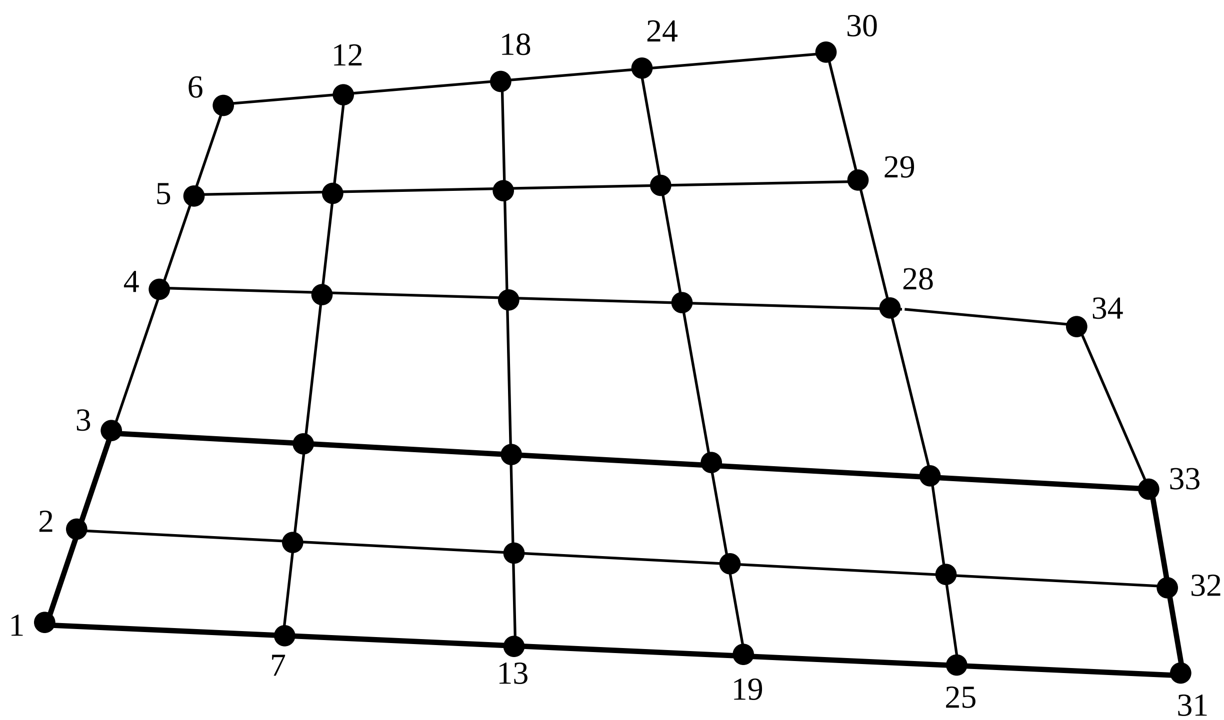
Grid #2 was developed in a similar manner except that the straight line generation option was also used to define the nodes lying along the material interface; i.e., along line 3 –> 33. In all cases the nodes along a line are equally spaced. In Section 3, only the following records are required to generate the mesh:

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | $x_1$ | $y_1$ | 0 | 1.0 | 0.0 | 0.0 |
| 31 | $x_{31}$ | $y_{31}$ | 6 | 1.0 | 0.0 | 0.0 |
| 33 | $x_{33}$ | $y_{33}$ | 1 | 1.0 | 0.0 | 0.0 |
| 3 | $x_3$ | $y_3$ | -6 | 1.0 | 0.0 | 0.0 |
| -1 | 0.0 | 0.0 | -1 | 1.0 | 0.0 | 0.0 |
| 34 | $x_{34}$ | $y_{34}$ | 0 | 1.0 | 0.0 | 0.0 |
| 28 | $x_{28}$ | $y_{28}$ | 0 | 1.0 | 0.0 | 0.0 |
| 30 | $x_{30}$ | $y_{30}$ | 1 | 1.0 | 0.0 | 0.0 |
| 6 | $x_6$ | $y_6$ | -6 | 1.0 | 0.0 | 0.0 |
| -3 | 0.0 | 0.0 | -1 | 1.0 | 0.0 | 0.0 |

Although the above discussion regarding nodal generation has dealt with four-node quadrilateral elements, the schemes described are equally applicable to 8- and/or 9-node elements.




(a) Grid #1




(b) Grid #2

**Figure 13**

Hypothetical Grids Prepared with the Aid of the Generation Options

{4} The material number **MNUM** must correspond to the appropriate material description given in Section 3.

{5} If the body being analyzed can be divided into a layer (or layers) of elements, and if the material number (**MNUM**) is the *same* for several elements within a layer, then the node point numbers associated with these elements can be simply established by means of the element data generation option. To generate a sequence of elements within a *single* layer, node point numbers are specified only for the first element, together with appropriate values for **NUMADD** and **INCADD** (**NUMLAY** and **INCLAY** are set equal to *zero*). For example, the bottom row of elements in the grid shown in Figure 13a could be established by entering the node numbers associated with element "a" and the values **NUMADD** = 4 and **INCADD** = 5. Assuming a material number equal to one (1), the input record corresponding to this specification would be:

```
1    6    7    2    1    4    5    0    0
```

Alternately, this bottom row of elements could be established by entering the node numbers associated with element "b" and the values **NUMADD** = 4 and **INCADD** = -5. Assuming once again a material number equal to one (1), the corresponding input record would be:

```
21   26   27   22   1    4    -5    0    0
```

In a similar manner, the left most column of elements in the grid could be established by entering the node numbers associated with element "a" and the values **NUMADD** = 2 and **INCADD** = 1. The corresponding input record would be:

```
1    6    7    2    1    2    1    0    0
```

If several layers of elements are made of the same material, the above generation option can be carried one step further. For example, the entire grid shown in Figure 13a could be established by entering the node numbers of element "a" and the values **NUMADD** = 4, **INCADD** = 5, **NUMLAY** = 2, and **INCLAY** = 1. The associated input record would thus be

```
1    6    7    2    1    4    5    2    1
```

where once again a material number equal to one (1) has been assumed. Alternately, the same grid could be generated by specifying the node points associated with element "a" and **NUMADD** = 2, **INCADD** = 1, **NUMLAY** = 4, and **INCLAY** = 5. The associated input record would thus be:

```
1    6    7    2    1    2    1    4    5
```

The grid generated with these two specifications would differ only in the order in which the elements were numbered by the TWOD program.

{6} The information supplied in this section allows the user to specify a distributed normal and/or shear traction acting along a straight or curved boundary. Nodal loads equivalent to the applied traction are then computed by the TWOD computer program.

{7} For three-node triangles or for four-node quadrilateral elements, a linearly varying (or constant) normal and/or shear traction distribution can be applied along a boundary.

{8} If a traction is specified over several nodes in a sequence (**NUM**, **NUM** + **INCR**, **NUM** + 2***INCR**, . . . , **NUMEND – INCR**, **NUMEND**), then this specification can be generated with a *single* input record. This is achieved by supplying, on this record, proper values for the quantities **NUM**, **NUMEND** and **INCR**. The quantity **NUMEND** denotes the number of the last node in the sequence; **INCR** represents the difference between the numbers of successive nodes.

{9} To be consistent with the order of node numbering employed in defining the element topology, during specification of the edge loading the nodes must likewise be specified in a *counterclockwise* sequence. A traction normal to an edge is assumed to be positive if it is directed *into* the element. A tangential traction is assumed to be positive if it acts in a *counterclockwise* direction with respect to the loaded element (Figure 14).

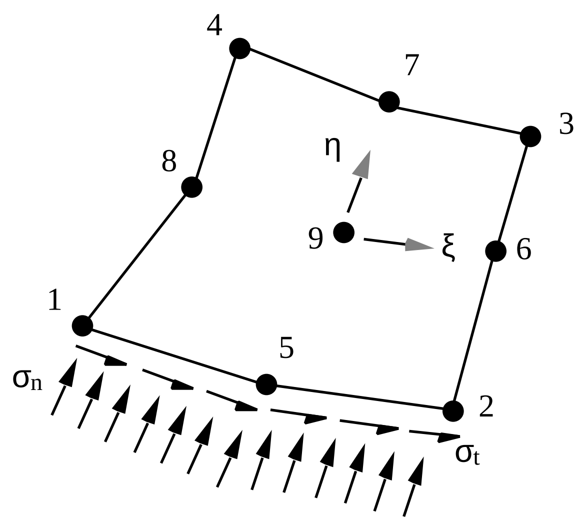



**Figure 14**
Normal and Tangential Distributed Edge Loads Applied to a Bi-Quadratic Lagrange Element

Such definitions are necessary in order to avoid confusion when distributed loads are specified along the interface between two elements, such as that shown in Figure 15. In this figure the edge loads, if they are assumed to act on element 1, are considered to be positive; if they are taken as acting on element 2, these same loads are negative.

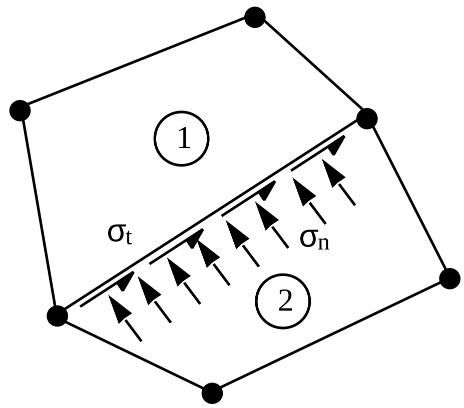



**Figure 15 :** Edge Loads Specified Along an Element Interface

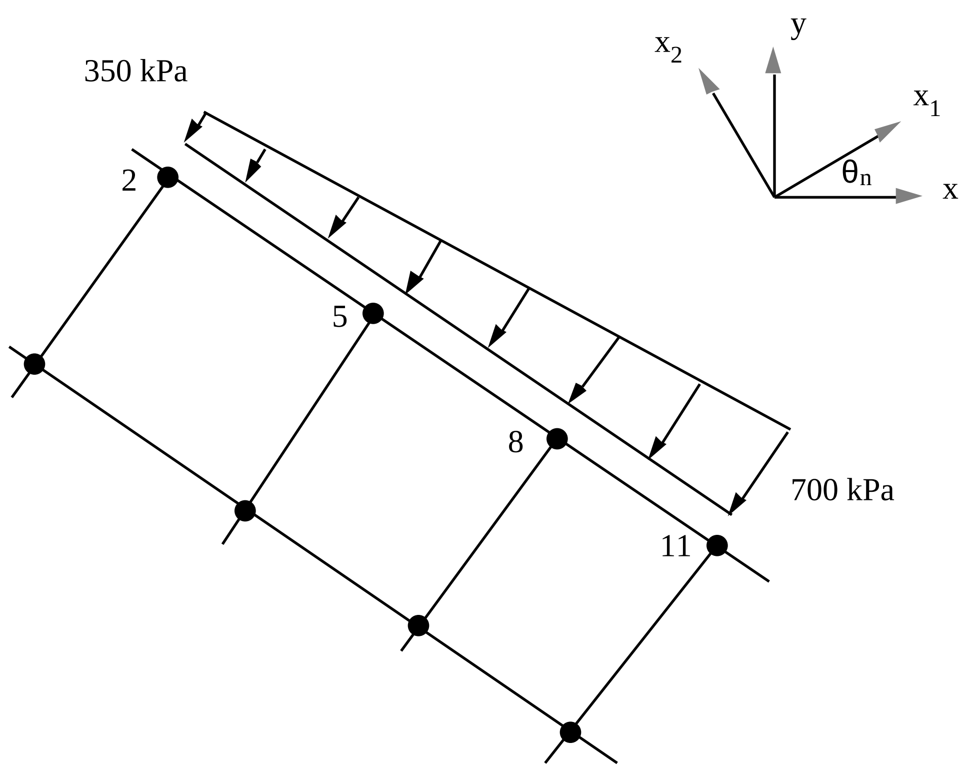



**Figure 16 :** Boundary with Normal Traction (Pressure) Applied

♦ EXAMPLE:
To specify the pressure loading along the boundary shown in Figure 16, the user would enter, in a single input record, the values **NUM** = 11, **NUMEND** = 2, **INCR** = -3, **PN(1)** = 700.00, **PT**(1) = 0.0, **PN(2)** = 350.0 and **PT**(2) = 0.0.

---

*NOTE*: If for a given node, **PN(1)** = **PN(2)** = 0 and **PT(1)** = **PT(2)** = 0, the node need not (and, for economy *should* not) be included in the distributed node point specifications.

{10} For eight- and/or for nine-node quadrilateral elements a parabolic or linearly varying (or constant) normal and/or shear traction distribution can be applied along a boundary.

{11} If a traction is specified over several nodes in a sequence (**NUM**, **NUM** + **INCR**, **NUM** + 2***INCR**, . . . , **NUMEND** – **INCR**, **NUMEND**), then this specification can be generated with a *single* input record. This is achieved by supplying, on this record, proper values for the quantities **NUM**, **NUMEND**, **INCR1** and **INCR2**. The quantity **NUMEND** denotes the number of the last *corner* node in the sequence; **INCR1** represents the difference between two successive corner node numbers; **INCR2** represents the difference between the number of a corner node and its adjacent mid-side node in the sequence .

{12} If a *linear* interpolation is to be used in specifying the tractions, **NEXTRA** should be set equal to zero (0). If, on the other hand, a *quadratic* interpolation is to be used in specifying the tractions, **NEXTRA** should be set equal to one (1). In the latter case, the quantities **XT** and **YT** represent the x- and y-coordinates of an intermediate point at which the intermediate value of traction, described by **PN(2)** and **PT(2)**, is known. This intermediate point need not coincide with any node point.

{13} The information supplied in this section allows the user to specify known applied displacements and/or loads at boundary and/or at interior nodes.

{14} If several node points in a sequence (**NUM**, **NUM** + **INCR**, **NUM** + 2***INCR**, . . . , **NUMEND** – **INCR**, **NUMEND**) all have identical specifications associated with them, these specifications can all be generated with a *single* input record. This is achieved by supplying, on this record, proper values for the quantities **NUM**, **NUMEND, INCR** and **THETA**. The quantity **NUMEND** denotes the number of the last node point in the sequence; **INCR** represents the difference between the numbers of successive node points; THETA represents the angle between the outward normal along the boundary and the global x-axis (see Figure 16).

{15} At interior node points, as well as along a boundary whose outward normal lies parallel to one of the coordinate directions, the applied displacements and/or loads are specified in terms of x-y components. Alternately, if nodal displacements and/or loads are specified along a boundary whose outward normal does *not* lie along one of the coordinate directions (i.e., $\theta_n \neq 0$, see Figure 16), then these specified quantities may be given in terms of *local* (rotated) $x_1$-$x_2$ components. If $\theta_n = 0$, the subscripts “1” and “2” appearing in this section refer to x and y, respectively, and IFLAG1 = IFLAGX, VALUE1 = VALUEX, IFLAG2 = IFLAGY, and VALUE2 = VALUEY.

{16} Concentrated (nodal) forces, acting in the $x_1$ – direction may be specified at any number of locations along the body by placing nodes at these locations. For each such node **IFLAG1** is set equal to zero (0) and **VALUE1** is set equal to the known value of applied force. Concentrated nodal forces acting in the positive $x_1$ direction are positive. Known values of displacement, in the $x_1$-direction, may be specified at any number of locations along the body by placing nodes at these locations. For each such node **IFLAG1** is set equal to one (1) and **VALUE1** is set equal to the known value of displacement.

{17} Concentrated (nodal) forces, acting in the $x_2$ – direction may be specified at any number of locations along the body by placing nodes at these locations. For each such node **IFLAG2** is set equal to zero (0) and **VALUE2** is set equal to the known value of applied force. Concentrated nodal forces acting in the positive $x_2$ direction are positive. Known values of displacement, in the $x_2$-direction, may be specified at any number of locations along the body by placing nodes at these locations. For each such node **IFLAG2** is set equal to one (1) and **VALUE2** is set equal to the known value of displacement.

*NOTE*: At node points where the $x_1$-direction displacement is unknown and no force is applied, **IFLAG1** should be set equal to zero (0) and **VALUE1** should set equal to 0.0 . Likewise, if at these node points the $x_2$-direction displacement is unknown and no force is applied, **IFLAG2** should be set equal to zero (0) and **VALUE2** should set equal to 0.0 . Since these values for **IFLAG1** , **VALUE1**, **IFLAG2** and

**VALUE2** are pre-set upon initiation of the program, *there is no need to supply nodal specification records for such cases*.

*NOTE*: If two or more contradictory displacement specifications are made for the same node point, the *first* of these is arbitrarily assumed to be correct. If two or more nodal forces are specified at the same node, their *sum* is used. If both a displacement and a force are specified for a node, the force – since it has no effect on the solution – is not printed.

---

♦ EXAMPLE:

For example, if pinned conditions were specified along the bottom boundary of the grid shown in Figure 13a, the appropriate input record would be

```
1   26  5  0.0   0  0.0  1  0.0
```

---

## V. PROGRAM OUTPUT

The output generated by TWOD consists of:

- an "echo" of the input data;
- the approximate nodal displacements. Intermediate values can of course be found by returning to the linear approximation for displacement within each element; and,
- the approximate values of the strains and stresses at the element integration points (the coordinates of these points are also printed).

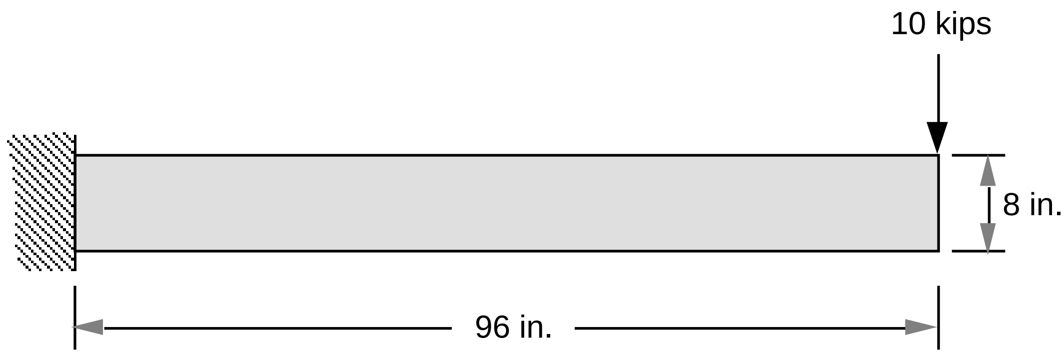
# VI. SAMPLE ANALYSES

The following analyses illustrate the correct manner in which to supply data to the TWOD computer program.

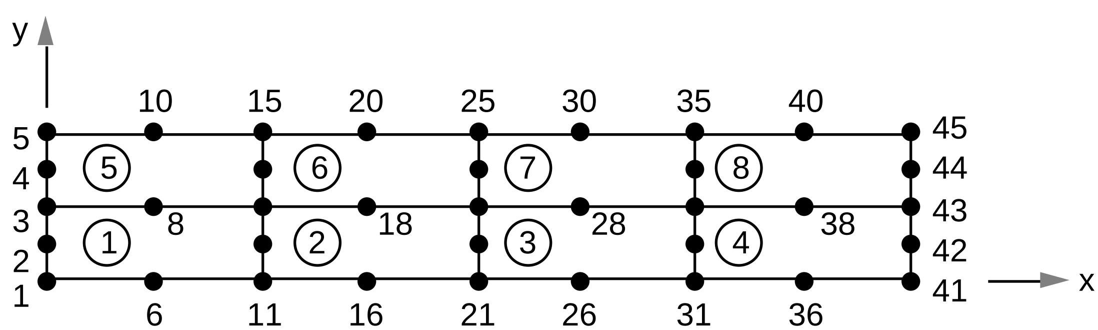
---

**Sample Analysis 1 :** bending simulation involving an anisotropic elastic beam.

This example involves the simulation of beam bending. The material is idealized as being anisotropic (orthotropic) elastic. A concentrated load is applied at the tip of the cantilevered beam. The physical model is shown below.




The following mesh is used to model the beam




A copy of the input data file associated with this sample analysis is given below. The results (output) associated with this analysis follow the input data.

```
TITLE
  plane stress analysis of an anisotropic elastic (timber) beam
  units of kips and inches
END
*
GENERAL
plane stress
analyze
END
```

```
*
MATERIAL
   1  2    0.0  0.0
   0.0
   2.30e+06  2.81e+04  2.81e+04  0.0
             1.05e+05  2.81e+04  0.0
                       1.05e+05  0.0
                                 1.80e+05
END
*
nodes
    1     0.0   0.0  0  1.0  0
   41    96.0   0.0  5  1.0  0
   45    96.0   8.0  1  1.0  0
    5     0.0   8.0 -5  1.0  0
   -1     0.0   0.0 -1  1.0  0
end
*
ELEMENTS
8-node quads
     1  11  13  3  6  12  8  2    1  1.0  3  10  1  2
end
END
*
CONCENTRATED SPECIFICATIONS
    1   5   1  0.0   1  0.0   1    0.0
   45  45   0  0.0   0  0.0   0  -10.0
END
*
finish
```

```
        twod : version 2.1 : 07-03-93




 plane stress analysis of an anisotropic elastic (timber) beam
 units of kips and inches


        --->  Plane  Stress  Analysis  <---



        MATERIAL   DATA:
        ===============

   * Material number   1          -> Type : anisotropic linear elastic
     ---------------

   angle between preferred element directions and global x-axis =    0.00 deg.

        C11 = 2.300E+06    C12 = 2.810E+04    C13 = 2.810E+04    C14 = 0.000E+00
                           C22 = 1.050E+05    C23 = 2.810E+04    C24 = 0.000E+00
                                              C33 = 1.050E+05    C34 = 0.000E+00
                                                                 C44 = 1.800E+05
                  X-direction body force = 0.000E+00
                  Y-direction body force = 0.000E+00




        NODE  POINT  INFORMATION:
        ========================

   node          x              y
  number     coordinate     coordinate
  ------     ----------     ----------

      1       0.000E+00      0.000E+00
      2      -3.553E-15      2.000E+00
      3       7.994E-15      4.000E+00
      4       0.000E+00      6.000E+00
      5       0.000E+00      8.000E+00
      6       1.200E+01      0.000E+00
      8       1.200E+01      4.000E+00
     10       1.200E+01      8.000E+00
     11       2.400E+01      0.000E+00
     12       2.400E+01      2.000E+00
     13       2.400E+01      4.000E+00
     14       2.400E+01      6.000E+00
     15       2.400E+01      8.000E+00
     16       3.600E+01      0.000E+00
```

```
      18       3.600E+01      4.000E+00
      20       3.600E+01      8.000E+00
      21       4.800E+01      0.000E+00
      22       4.800E+01      2.000E+00
      23       4.800E+01      4.000E+00
      24       4.800E+01      6.000E+00
      25       4.800E+01      8.000E+00
      26       6.000E+01      0.000E+00
      28       6.000E+01      4.000E+00
      30       6.000E+01      8.000E+00
      31       7.200E+01      0.000E+00
      32       7.200E+01      2.000E+00
      33       7.200E+01      4.000E+00
      34       7.200E+01      6.000E+00
      35       7.200E+01      8.000E+00
      36       8.400E+01      0.000E+00
      38       8.400E+01      4.000E+00
      40       8.400E+01      8.000E+00
      41       9.600E+01      0.000E+00
      42       9.600E+01      2.000E+00
      43       9.600E+01      4.000E+00
      44       9.600E+01      6.000E+00
      45       9.600E+01      8.000E+00



                         ELEMENT  INFORMATION:
                         ====================


--> eight-node quadrilateral data:
    ----------------------------

element  node  node  node  node  node  node  node  node       material
 number   #1    #2    #3    #4    #5    #6    #7    #8         number      area
-------  ----  ----  ----  ----  ----  ----  ----  ----       --------     ----

    1  :   1    11    13     3     6    12     8     2            1      1.000E+00
    2  :  11    21    23    13    16    22    18    12            1      1.000E+00
    3  :  21    31    33    23    26    32    28    22            1      1.000E+00
    4  :  31    41    43    33    36    42    38    32            1      1.000E+00
    5  :   3    13    15     5     8    14    10     4            1      1.000E+00
    6  :  13    23    25    15    18    24    20    14            1      1.000E+00
    7  :  23    33    35    25    28    34    30    24            1      1.000E+00
    8  :  33    43    45    35    38    44    40    34            1      1.000E+00


              NODAL   SPECIFICATIONS:
              ======================
   node
  number                          specifications
```

```
  ------                        -----------------

      1    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
      2    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
      3    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
      4    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
      5    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
     45    force-X =  0.000E+00    force-Y = -1.000E+01   angle =   0.00


OUTPUT   DATA:
=============


                             Element Strains and Stresses
                             ============================


                             eight-node quadrilateral data:
                             *****************************

    Xc       eps-X      eps-Y      eps-Z     gam-XY      eps-1      eps-2
    Yc       sig-X      sig-Y      sig-Z     tau-XY      sig-1      sig-2    angle
 --------  ---------  ---------  ---------  ---------  ---------  ---------  -----

--> element no.   1 :

 5.07E+00 -2.911E-05  2.671E-06  7.076E-06 -6.552E-06  3.968E-06 -3.041E-05
 8.45E-01 -6.668E+01 -3.387E-01  0.000E+00 -1.179E+00 -3.178E-01 -6.670E+01 -88.98

 1.89E+01 -2.522E-05  5.490E-06  5.279E-06 -3.861E-06  5.968E-06 -2.569E-05
 8.45E-01 -5.769E+01  1.627E-02  0.000E+00 -6.949E-01  2.464E-02 -5.770E+01 -89.31

 5.07E+00 -7.670E-06  1.656E-06  1.610E-06 -7.336E-06  5.686E-06 -1.170E-05
 3.15E+00 -1.755E+01  3.561E-03  0.000E+00 -1.321E+00  1.023E-01 -1.765E+01 -85.72

 1.89E+01 -6.626E-06  1.703E-06  1.318E-06 -1.003E-05  8.395E-06 -1.332E-05
 3.15E+00 -1.516E+01  2.965E-02  0.000E+00 -1.805E+00  2.412E-01 -1.537E+01 -83.31

--> element no.   2 :

 2.91E+01 -2.134E-05  4.843E-06  4.415E-06 -3.476E-06  5.297E-06 -2.180E-05
 8.45E-01 -4.883E+01  3.288E-02  0.000E+00 -6.258E-01  4.089E-02 -4.883E+01 -89.27

 4.29E+01 -1.743E-05  3.975E-06  3.601E-06 -3.911E-06  4.667E-06 -1.812E-05
 8.45E-01 -3.987E+01  2.881E-02  0.000E+00 -7.040E-01  4.122E-02 -3.989E+01 -88.99

 2.91E+01 -5.750E-06  5.448E-07  1.393E-06 -1.043E-05  8.289E-06 -1.349E-05
 3.15E+00 -1.317E+01 -6.522E-02  0.000E+00 -1.877E+00  1.983E-01 -1.343E+01 -82.01

 4.29E+01 -4.690E-06  1.054E-06  9.732E-07 -9.909E-06  8.498E-06 -1.213E-05
 3.15E+00 -1.073E+01  6.199E-03  0.000E+00 -1.784E+00  2.947E-01 -1.102E+01 -80.81
```

```
--> element no.   3 :

 5.31E+01 -1.357E-05  2.674E-06  2.917E-06 -4.138E-06  3.667E-06 -1.457E-05
 8.45E-01 -3.106E+01 -1.863E-02  0.000E+00 -7.448E-01 -7.657E-04 -3.108E+01 -88.63

 6.69E+01 -9.731E-06  1.457E-06  2.214E-06 -3.493E-06  2.458E-06 -1.073E-05
 8.45E-01 -2.228E+01 -5.820E-02  0.000E+00 -6.288E-01 -4.042E-02 -2.230E+01 -88.38

 5.31E+01 -3.666E-06  7.631E-07  7.770E-07 -9.956E-06  8.748E-06 -1.165E-05
 3.15E+00 -8.389E+00 -1.070E-03  0.000E+00 -1.792E+00  3.658E-01 -8.756E+00 -78.43

 6.69E+01 -2.528E-06  9.289E-07  4.280E-07 -9.642E-06  8.996E-06 -1.059E-05
 3.15E+00 -5.777E+00  3.851E-02  0.000E+00 -1.735E+00  5.171E-01 -6.255E+00 -74.58

--> element no.   4 :

 7.71E+01 -5.698E-06  2.106E-06  9.612E-07 -4.828E-06  4.411E-06 -8.003E-06
 8.45E-01 -1.302E+01  8.803E-02  0.000E+00 -8.690E-01  1.454E-01 -1.308E+01 -86.22

 9.09E+01 -1.666E-06 -3.920E-07  5.506E-07 -3.387E-06  2.418E-06 -4.476E-06
 8.45E-01 -3.826E+00 -7.249E-02  0.000E+00 -6.097E-01  2.407E-02 -3.923E+00 -81.00

 7.71E+01 -1.692E-06  1.185E-06  1.359E-07 -1.079E-05  1.063E-05 -1.114E-05
 3.15E+00 -3.856E+00  8.064E-02  0.000E+00 -1.941E+00  8.771E-01 -4.652E+00 -67.70

 9.09E+01 -6.707E-07 -3.770E-06  1.188E-06 -9.383E-06  7.290E-06 -1.173E-05
 3.15E+00 -1.615E+00 -3.813E-01  0.000E+00 -1.689E+00  7.999E-01 -2.796E+00 -55.03

--> element no.   5 :

 5.07E+00  7.670E-06 -1.660E-06 -1.608E-06 -7.323E-06  1.169E-05 -5.678E-06
 4.85E+00  1.755E+01 -3.976E-03  0.000E+00 -1.318E+00  1.765E+01 -1.024E-01  -4.27

 1.89E+01  6.627E-06 -1.691E-06 -1.321E-06 -9.981E-06  1.328E-05 -8.345E-06
 4.85E+00  1.516E+01 -2.848E-02  0.000E+00 -1.797E+00  1.537E+01 -2.381E-01  -6.66

 5.07E+00  2.911E-05 -2.681E-06 -7.073E-06 -6.567E-06  3.041E-05 -3.984E-06
 7.15E+00  6.668E+01  3.377E-01  0.000E+00 -1.182E+00  6.670E+01  3.167E-01  -1.02

 1.89E+01  2.521E-05 -5.501E-06 -5.276E-06 -3.910E-06  2.570E-05 -5.991E-06
 7.15E+00  5.769E+01 -1.732E-02  0.000E+00 -7.038E-01  5.770E+01 -2.590E-02  -0.70

--> element no.   6 :

 2.91E+01  5.746E-06 -5.198E-07 -1.399E-06 -1.042E-05  1.349E-05 -8.268E-06
 4.85E+00  1.316E+01  6.758E-02  0.000E+00 -1.876E+00  1.343E+01 -1.958E-01  -7.99

 4.29E+01  4.704E-06 -1.141E-06 -9.535E-07 -1.016E-05  1.236E-05 -8.794E-06
 4.85E+00  1.076E+01 -1.442E-02  0.000E+00 -1.830E+00  1.106E+01 -3.166E-01  -9.38

 2.91E+01  2.134E-05 -4.810E-06 -4.425E-06 -3.455E-06  2.179E-05 -5.259E-06
 7.15E+00  4.883E+01 -2.960E-02  0.000E+00 -6.219E-01  4.884E+01 -3.752E-02  -0.73
```

```
 4.29E+01  1.742E-05 -3.955E-06 -3.602E-06 -3.794E-06  1.807E-05 -4.609E-06
 7.15E+00  3.984E+01 -2.713E-02  0.000E+00 -6.829E-01  3.986E+01 -3.883E-02  -0.98

--> element no.   7 :

 5.31E+01  3.627E-06 -6.505E-07 -7.967E-07 -9.780E-06  1.150E-05 -8.523E-06
 4.85E+00  8.302E+00  1.124E-02  0.000E+00 -1.760E+00  8.660E+00 -3.471E-01 -11.50

 6.69E+01  2.666E-06 -2.067E-06 -1.604E-07 -9.503E-06  1.009E-05 -9.494E-06
 4.85E+00  6.069E+00 -1.466E-01  0.000E+00 -1.711E+00  6.509E+00 -5.862E-01 -14.41

 5.31E+01  1.361E-05 -2.507E-06 -2.971E-06 -3.904E-06  1.450E-05 -3.403E-06
 7.15E+00  3.115E+01  3.568E-02  0.000E+00 -7.027E-01  3.116E+01  1.982E-02  -1.29

 6.69E+01  9.615E-06 -2.783E-06 -1.828E-06 -5.140E-06  1.147E-05 -4.637E-06
 7.15E+00  2.199E+01 -7.339E-02  0.000E+00 -9.252E-01  2.202E+01 -1.121E-01  -2.40

--> element no.   8 :

 7.71E+01  1.317E-06  3.839E-06 -1.380E-06 -1.014E-05  1.280E-05 -7.644E-06
 4.85E+00  3.097E+00  4.013E-01  0.000E+00 -1.826E+00  4.019E+00 -5.203E-01 -26.78

 9.09E+01  2.679E-07 -8.576E-06  2.223E-06 -1.013E-05  6.900E-06 -1.521E-05
 4.85E+00  4.377E-01 -8.304E-01  0.000E+00 -1.824E+00  1.734E+00 -2.127E+00 -35.41

 7.71E+01  5.985E-06  2.699E-06 -2.324E-06 -2.020E-06  6.946E-06  1.738E-06
 7.15E+00  1.378E+01  3.862E-01  0.000E+00 -3.637E-01  1.379E+01  3.764E-01  -1.55

 9.09E+01  2.291E-06 -1.201E-05  2.602E-06 -4.876E-06  3.795E-06 -1.352E-05
 7.15E+00  5.004E+00 -1.124E+00  0.000E+00 -8.778E-01  5.127E+00 -1.247E+00  -7.99
```

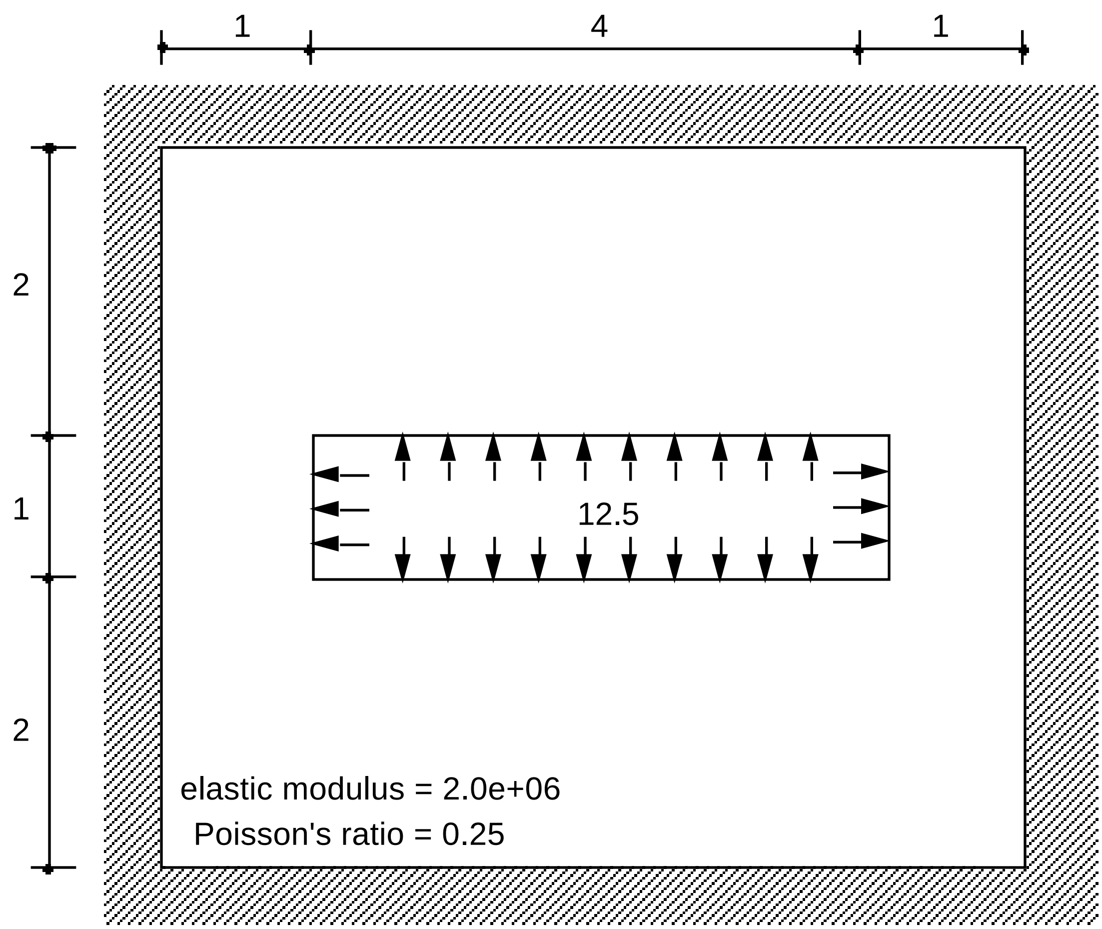
```
OUTPUT   DATA:
=============

                    Coordinates                     Displacements
     node           -----------                     -------------
    number             X           Y                   U              V
    ------       ---------   ---------          ---------      ---------

       1         0.000E+00   0.000E+00         -7.736E-25     -1.097E-25
       2        -3.553E-15   2.000E+00         -5.146E-25      4.339E-27
       3         7.994E-15   4.000E+00          6.000E-28     -8.776E-26
       4         0.000E+00   6.000E+00          5.141E-25      4.210E-27
       5         0.000E+00   8.000E+00          7.743E-25     -1.095E-25
       6         1.200E+01   0.000E+00         -4.417E-04     -7.674E-04
       8         1.200E+01   4.000E+00         -1.591E-08     -7.526E-04
      10         1.200E+01   8.000E+00          4.416E-04     -7.674E-04
      11         2.400E+01   0.000E+00         -8.320E-04     -2.773E-03
      12         2.400E+01   2.000E+00         -4.096E-04     -2.764E-03
      13         2.400E+01   4.000E+00         -2.459E-08     -2.763E-03
```

| | | | | |
|---|---|---|---|---|
| 14 | 2.400E+01 | 6.000E+00 | 4.096E-04 | -2.764E-03 |
| 15 | 2.400E+01 | 8.000E+00 | 8.319E-04 | -2.773E-03 |
| 16 | 3.600E+01 | 0.000E+00 | -1.152E-03 | -5.845E-03 |
| 18 | 3.600E+01 | 4.000E+00 | 4.700E-08 | -5.834E-03 |
| 20 | 3.600E+01 | 8.000E+00 | 1.152E-03 | -5.845E-03 |
| 21 | 4.800E+01 | 0.000E+00 | -1.421E-03 | -9.799E-03 |
| 22 | 4.800E+01 | 2.000E+00 | -7.054E-04 | -9.792E-03 |
| 23 | 4.800E+01 | 4.000E+00 | 2.727E-07 | -9.790E-03 |
| 24 | 4.800E+01 | 6.000E+00 | 7.053E-04 | -9.792E-03 |
| 25 | 4.800E+01 | 8.000E+00 | 1.421E-03 | -9.799E-03 |
| 26 | 6.000E+01 | 0.000E+00 | -1.623E-03 | -1.446E-02 |
| 28 | 6.000E+01 | 4.000E+00 | -3.456E-07 | -1.445E-02 |
| 30 | 6.000E+01 | 8.000E+00 | 1.623E-03 | -1.446E-02 |
| 31 | 7.200E+01 | 0.000E+00 | -1.776E-03 | -1.965E-02 |
| 32 | 7.200E+01 | 2.000E+00 | -8.822E-04 | -1.965E-02 |
| 33 | 7.200E+01 | 4.000E+00 | 5.809E-07 | -1.964E-02 |
| 34 | 7.200E+01 | 6.000E+00 | 8.828E-04 | -1.965E-02 |
| 35 | 7.200E+01 | 8.000E+00 | 1.773E-03 | -1.966E-02 |
| 36 | 8.400E+01 | 0.000E+00 | -1.857E-03 | -2.520E-02 |
| 38 | 8.400E+01 | 4.000E+00 | -1.638E-06 | -2.520E-02 |
| 40 | 8.400E+01 | 8.000E+00 | 1.864E-03 | -2.519E-02 |
| 41 | 9.600E+01 | 0.000E+00 | -1.885E-03 | -3.089E-02 |
| 42 | 9.600E+01 | 2.000E+00 | -9.411E-04 | -3.089E-02 |
| 43 | 9.600E+01 | 4.000E+00 | -4.688E-06 | -3.091E-02 |
| 44 | 9.600E+01 | 6.000E+00 | 9.394E-04 | -3.096E-02 |
| 45 | 9.600E+01 | 8.000E+00 | 1.907E-03 | -3.102E-02 |

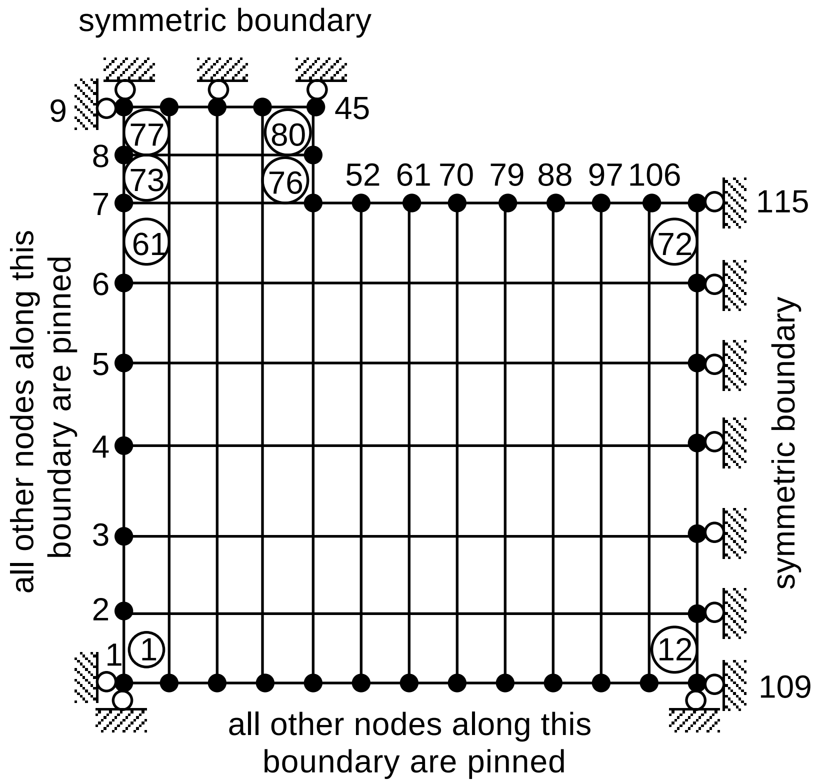
```
>>> END-OF-FILE encountered during read <<<
```

**Sample Analysis 2:** rectangular solid with a rectangular cut-out and internal pressure applied.

This example involves the analysis of a rectangular elastic solid with a rectangular cut-out. The solid is subjected to an internal pressure. The material is idealized as being isotropic elastic. The physical model is shown below.




Due to the double symmetry of the geometry, of the loading, and of the boundary conditions, only a quarter of the body needs to be analyzed. As such, the following mesh is used to model the beam




A copy of the input data file associated with this sample analysis is given below. The results (output) associated with this analysis follow the input data.

```
TITLE
  plane strain analysis of long bar with a rectangular cut-out
END
*
GENERAL
plane strain
analyze
END
*
MATERIAL
   1  1    0.0  0.0
   2.0e+06  0.25
END
*
nodes
     1     0.0   0.0  0  1.0  0
   109     3.0   0.0  9  1.0  0
   115     3.0   2.0  1  1.0  0
     7     0.0   2.0 -9  1.0  0
     9     0.0   2.5  1  1.0  0
    45     1.0   2.5  9  1.0  0
    43     1.0   2.0 -1  1.0  0
```

```
end
*
ELEMENTS
4-node quads
     1  10  11  2  1  1.0  11  9  5  1
    16  17   8  7  1  1.0   3  9  1  1
end
END
*
DISTRIBUTED SPECS
4-node quads
   115  43  -9  12.5  0.0  12.5  0.0
    43  45   1  12.5  0.0  12.5  0.0
end
END
*
CONCENTRATED SPECIFICATIONS
    1 109   9  0.0   1  0.0   1   0.0
  109 115   1  0.0   1  0.0   0   0.0
    1   9   1  0.0   1  0.0   1   0.0
    9  45   9  0.0   0  0.0   1   0.0
END
*
finish
```

```
twod : version 2.2 : 19-05-94


plane strain analysis of long bar with a rectangular cut-out


       --->  Plane  Strain  Analysis  <---



       MATERIAL   DATA:
       ===============

  * Material number   1          -> Type : isotropic linear elastic
    ---------------

                  Modulus of Elasticity = 2.000E+06
                        Poisson's ratio = 0.250

                 X-direction body force = 0.000E+00
                 Y-direction body force = 0.000E+00



       NODE  POINT  INFORMATION:
       ========================

  node          x              y
 number     coordinate     coordinate
 ------     ----------     ----------

     1       0.000E+00      0.000E+00
     2       4.163E-16      3.333E-01
     3       1.490E-15      6.667E-01
     4       5.829E-16      1.000E+00
     5      -8.882E-16      1.333E+00
     6       0.000E+00      1.667E+00
     7       0.000E+00      2.000E+00
     8       0.000E+00      2.250E+00
     9       0.000E+00      2.500E+00
    10       2.500E-01      0.000E+00
    11       2.500E-01      3.333E-01
    12       2.500E-01      6.667E-01
    13       2.500E-01      1.000E+00
    14       2.500E-01      1.333E+00
    15       2.500E-01      1.667E+00
    16       2.500E-01      2.000E+00
    17       2.500E-01      2.250E+00
    18       2.500E-01      2.500E+00
    19       5.000E-01      0.000E+00
    20       5.000E-01      3.333E-01
    21       5.000E-01      6.667E-01
    22       5.000E-01      1.000E+00
```

| | | |
|---|---|---|
| 23 | 5.000E-01 | 1.333E+00 |
| 24 | 5.000E-01 | 1.667E+00 |
| 25 | 5.000E-01 | 2.000E+00 |
| 26 | 5.000E-01 | 2.250E+00 |
| 27 | 5.000E-01 | 2.500E+00 |
| 28 | 7.500E-01 | 0.000E+00 |
| 29 | 7.500E-01 | 3.333E-01 |
| 30 | 7.500E-01 | 6.667E-01 |
| 31 | 7.500E-01 | 1.000E+00 |
| 32 | 7.500E-01 | 1.333E+00 |
| 33 | 7.500E-01 | 1.667E+00 |
| 34 | 7.500E-01 | 2.000E+00 |
| 35 | 7.500E-01 | 2.250E+00 |
| 36 | 7.500E-01 | 2.500E+00 |
| 37 | 1.000E+00 | 0.000E+00 |
| 38 | 1.000E+00 | 3.333E-01 |
| 39 | 1.000E+00 | 6.667E-01 |
| 40 | 1.000E+00 | 1.000E+00 |
| 41 | 1.000E+00 | 1.333E+00 |
| 42 | 1.000E+00 | 1.667E+00 |
| 43 | 1.000E+00 | 2.000E+00 |
| 44 | 1.000E+00 | 2.250E+00 |
| 45 | 1.000E+00 | 2.500E+00 |
| 46 | 1.250E+00 | 0.000E+00 |
| 47 | 1.250E+00 | 3.333E-01 |
| 48 | 1.250E+00 | 6.667E-01 |
| 49 | 1.250E+00 | 1.000E+00 |
| 50 | 1.250E+00 | 1.333E+00 |
| 51 | 1.250E+00 | 1.667E+00 |
| 52 | 1.250E+00 | 2.000E+00 |
| 55 | 1.500E+00 | 0.000E+00 |
| 56 | 1.500E+00 | 3.333E-01 |
| 57 | 1.500E+00 | 6.667E-01 |
| 58 | 1.500E+00 | 1.000E+00 |
| 59 | 1.500E+00 | 1.333E+00 |
| 60 | 1.500E+00 | 1.667E+00 |
| 61 | 1.500E+00 | 2.000E+00 |
| 64 | 1.750E+00 | 0.000E+00 |
| 65 | 1.750E+00 | 3.333E-01 |
| 66 | 1.750E+00 | 6.667E-01 |
| 67 | 1.750E+00 | 1.000E+00 |
| 68 | 1.750E+00 | 1.333E+00 |
| 69 | 1.750E+00 | 1.667E+00 |
| 70 | 1.750E+00 | 2.000E+00 |
| 73 | 2.000E+00 | 0.000E+00 |
| 74 | 2.000E+00 | 3.333E-01 |
| 75 | 2.000E+00 | 6.667E-01 |
| 76 | 2.000E+00 | 1.000E+00 |
| 77 | 2.000E+00 | 1.333E+00 |
| 78 | 2.000E+00 | 1.667E+00 |
| 79 | 2.000E+00 | 2.000E+00 |
| 82 | 2.250E+00 | 0.000E+00 |
| 83 | 2.250E+00 | 3.333E-01 |

| | | |
|---|---|---|
| 84 | 2.250E+00 | 6.667E-01 |
| 85 | 2.250E+00 | 1.000E+00 |
| 86 | 2.250E+00 | 1.333E+00 |
| 87 | 2.250E+00 | 1.667E+00 |
| 88 | 2.250E+00 | 2.000E+00 |
| 91 | 2.500E+00 | 0.000E+00 |
| 92 | 2.500E+00 | 3.333E-01 |
| 93 | 2.500E+00 | 6.667E-01 |
| 94 | 2.500E+00 | 1.000E+00 |
| 95 | 2.500E+00 | 1.333E+00 |
| 96 | 2.500E+00 | 1.667E+00 |
| 97 | 2.500E+00 | 2.000E+00 |
| 100 | 2.750E+00 | 0.000E+00 |
| 101 | 2.750E+00 | 3.333E-01 |
| 102 | 2.750E+00 | 6.667E-01 |
| 103 | 2.750E+00 | 1.000E+00 |
| 104 | 2.750E+00 | 1.333E+00 |
| 105 | 2.750E+00 | 1.667E+00 |
| 106 | 2.750E+00 | 2.000E+00 |
| 109 | 3.000E+00 | 0.000E+00 |
| 110 | 3.000E+00 | 3.333E-01 |
| 111 | 3.000E+00 | 6.667E-01 |
| 112 | 3.000E+00 | 1.000E+00 |
| 113 | 3.000E+00 | 1.333E+00 |
| 114 | 3.000E+00 | 1.667E+00 |
| 115 | 3.000E+00 | 2.000E+00 |

ELEMENT INFORMATION:
====================

--> four-node quadrilateral data:
----------------------------

| element number | | node #1 | node #2 | node #3 | node #4 | material number | area |
|---|---|---|---|---|---|---|---|
| 1 | : | 1 | 10 | 11 | 2 | 1 | 1.000E+00 |
| 2 | : | 10 | 19 | 20 | 11 | 1 | 1.000E+00 |
| 3 | : | 19 | 28 | 29 | 20 | 1 | 1.000E+00 |
| 4 | : | 28 | 37 | 38 | 29 | 1 | 1.000E+00 |
| 5 | : | 37 | 46 | 47 | 38 | 1 | 1.000E+00 |
| 6 | : | 46 | 55 | 56 | 47 | 1 | 1.000E+00 |
| 7 | : | 55 | 64 | 65 | 56 | 1 | 1.000E+00 |
| 8 | : | 64 | 73 | 74 | 65 | 1 | 1.000E+00 |
| 9 | : | 73 | 82 | 83 | 74 | 1 | 1.000E+00 |
| 10 | : | 82 | 91 | 92 | 83 | 1 | 1.000E+00 |

```
 11  :  91   100   101    92                                          1      1.000E+00
 12  : 100   109   110   101                                          1      1.000E+00
 13  :   2    11    12     3                                          1      1.000E+00
 14  :  11    20    21    12                                          1      1.000E+00
 15  :  20    29    30    21                                          1      1.000E+00
 16  :  29    38    39    30                                          1      1.000E+00
 17  :  38    47    48    39                                          1      1.000E+00
 18  :  47    56    57    48                                          1      1.000E+00
 19  :  56    65    66    57                                          1      1.000E+00
 20  :  65    74    75    66                                          1      1.000E+00
 21  :  74    83    84    75                                          1      1.000E+00
 22  :  83    92    93    84                                          1      1.000E+00
 23  :  92   101   102    93                                          1      1.000E+00
 24  : 101   110   111   102                                          1      1.000E+00
 25  :   3    12    13     4                                          1      1.000E+00
 26  :  12    21    22    13                                          1      1.000E+00
 27  :  21    30    31    22                                          1      1.000E+00
 28  :  30    39    40    31                                          1      1.000E+00
 29  :  39    48    49    40                                          1      1.000E+00
 30  :  48    57    58    49                                          1      1.000E+00
 31  :  57    66    67    58                                          1      1.000E+00
 32  :  66    75    76    67                                          1      1.000E+00
 33  :  75    84    85    76                                          1      1.000E+00
 34  :  84    93    94    85                                          1      1.000E+00
 35  :  93   102   103    94                                          1      1.000E+00
 36  : 102   111   112   103                                          1      1.000E+00
 37  :   4    13    14     5                                          1      1.000E+00
 38  :  13    22    23    14                                          1      1.000E+00
 39  :  22    31    32    23                                          1      1.000E+00
 40  :  31    40    41    32                                          1      1.000E+00
 41  :  40    49    50    41                                          1      1.000E+00
 42  :  49    58    59    50                                          1      1.000E+00
 43  :  58    67    68    59                                          1      1.000E+00
 44  :  67    76    77    68                                          1      1.000E+00
 45  :  76    85    86    77                                          1      1.000E+00
 46  :  85    94    95    86                                          1      1.000E+00
 47  :  94   103   104    95                                          1      1.000E+00
 48  : 103   112   113   104                                          1      1.000E+00
 49  :   5    14    15     6                                          1      1.000E+00
 50  :  14    23    24    15                                          1      1.000E+00
 51  :  23    32    33    24                                          1      1.000E+00
 52  :  32    41    42    33                                          1      1.000E+00
 53  :  41    50    51    42                                          1      1.000E+00
 54  :  50    59    60    51                                          1      1.000E+00
 55  :  59    68    69    60                                          1      1.000E+00
 56  :  68    77    78    69                                          1      1.000E+00
 57  :  77    86    87    78                                          1      1.000E+00
 58  :  86    95    96    87                                          1      1.000E+00
 59  :  95   104   105    96                                          1      1.000E+00
 60  : 104   113   114   105                                          1      1.000E+00
 61  :   6    15    16     7                                          1      1.000E+00
 62  :  15    24    25    16                                          1      1.000E+00
 63  :  24    33    34    25                                          1      1.000E+00
```

```
64  :  33    42    43    34                                                  1      1.000E+00
65  :  42    51    52    43                                                  1      1.000E+00
66  :  51    60    61    52                                                  1      1.000E+00
67  :  60    69    70    61                                                  1      1.000E+00
68  :  69    78    79    70                                                  1      1.000E+00
69  :  78    87    88    79                                                  1      1.000E+00
70  :  87    96    97    88                                                  1      1.000E+00
71  :  96   105   106    97                                                  1      1.000E+00
72  : 105   114   115   106                                                  1      1.000E+00
73  :  16    17     8     7                                                  1      1.000E+00
74  :  25    26    17    16                                                  1      1.000E+00
75  :  34    35    26    25                                                  1      1.000E+00
76  :  43    44    35    34                                                  1      1.000E+00
77  :  17    18     9     8                                                  1      1.000E+00
78  :  26    27    18    17                                                  1      1.000E+00
79  :  35    36    27    26                                                  1      1.000E+00
80  :  44    45    36    35                                                  1      1.000E+00


              NODAL   SPECIFICATIONS:
              ======================

  node
 number                             specifications
 ------                           ------------------

   115    force-X =  0.000E+00    force-Y = -1.562E+00   angle =   0.00
   106    force-X =  0.000E+00    force-Y = -3.125E+00   angle =   0.00
    97    force-X =  0.000E+00    force-Y = -3.125E+00   angle =   0.00
    88    force-X =  0.000E+00    force-Y = -3.125E+00   angle =   0.00
    79    force-X =  0.000E+00    force-Y = -3.125E+00   angle =   0.00
    70    force-X =  0.000E+00    force-Y = -3.125E+00   angle =   0.00
    61    force-X =  0.000E+00    force-Y = -3.125E+00   angle =   0.00
    52    force-X =  0.000E+00    force-Y = -3.125E+00   angle =   0.00
    43    force-X =  0.000E+00    force-Y = -1.562E+00   angle =   0.00
    43    force-X = -1.562E+00    force-Y =  0.000E+00   angle =   0.00
    44    force-X = -3.125E+00    force-Y =  0.000E+00   angle =   0.00
    45    force-X = -1.562E+00    force-Y =  0.000E+00   angle =   0.00
     1    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
    10    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
    19    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
    28    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
    37    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
    46    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
    55    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
    64    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
    73    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
    82    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
    91    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
   100    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
   109    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
   109    displ-X =  0.000E+00    force-Y =  0.000E+00   angle =   0.00
```

```
     110    displ-X =  0.000E+00    force-Y =  0.000E+00   angle =   0.00
     111    displ-X =  0.000E+00    force-Y =  0.000E+00   angle =   0.00
     112    displ-X =  0.000E+00    force-Y =  0.000E+00   angle =   0.00
     113    displ-X =  0.000E+00    force-Y =  0.000E+00   angle =   0.00
     114    displ-X =  0.000E+00    force-Y =  0.000E+00   angle =   0.00
     115    displ-X =  0.000E+00    force-Y =  0.000E+00   angle =   0.00
       1    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
       2    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
       3    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
       4    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
       5    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
       6    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
       7    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
       8    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
       9    displ-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
       9    force-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
      18    force-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
      27    force-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
      36    force-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00
      45    force-X =  0.000E+00    displ-Y =  0.000E+00   angle =   0.00


 OUTPUT   DATA:
 =============


                              Element Strains and Stresses
                              ============================


                              four-node quadrilateral data:
                              ****************************

    Xc       eps-X      eps-Y      eps-Z     gam-XY      eps-1      eps-2
    Yc       sig-X      sig-Y      sig-Z     tau-XY      sig-1      sig-2    angle
 --------  ---------  ---------  ---------  ---------  ---------  ---------  -----

--> element no.   1 :

 1.25E-01 -4.846E-07 -3.344E-07  0.000E+00 -8.093E-07  4.033E-07 -1.222E-06
 1.67E-01 -1.431E+00 -1.190E+00 -6.552E-01 -6.475E-01 -6.519E-01 -1.969E+00 -50.26

--> element no.   2 :

 3.75E-01 -3.508E-07 -8.883E-07  0.000E+00 -1.283E-06  6.909E-07 -1.930E-06
 1.67E-01 -1.553E+00 -2.413E+00 -9.913E-01 -1.026E+00 -8.700E-01 -3.095E+00 -33.63

--> element no.   3 :

 6.25E-01 -2.672E-07 -1.331E-06  0.000E+00 -1.750E-06  1.031E-06 -2.628E-06
 1.67E-01 -1.706E+00 -3.407E+00 -1.278E+00 -1.400E+00 -9.179E-01 -4.195E+00 -29.36
```

```
--> element no.   4 :

 8.75E-01 -1.849E-07 -1.797E-06  0.000E+00 -2.117E-06  1.275E-06 -3.257E-06
 1.67E-01 -1.881E+00 -4.461E+00 -1.586E+00 -1.694E+00 -1.042E+00 -5.300E+00 -26.36

--> element no.   5 :

 1.12E+00 -9.219E-08 -2.302E-06  0.000E+00 -2.349E-06  1.399E-06 -3.793E-06
 1.67E-01 -2.063E+00 -5.599E+00 -1.916E+00 -1.879E+00 -1.251E+00 -6.411E+00 -23.37

--> element no.   6 :

 1.38E+00  7.841E-09 -2.828E-06  0.000E+00 -2.416E-06  1.391E-06 -4.211E-06
 1.67E-01 -2.243E+00 -6.780E+00 -2.256E+00 -1.932E+00 -1.532E+00 -7.492E+00 -20.21

--> element no.   7 :

 1.62E+00  1.022E-07 -3.338E-06  0.000E+00 -2.310E-06  1.262E-06 -4.498E-06
 1.67E-01 -2.425E+00 -7.930E+00 -2.589E+00 -1.848E+00 -1.862E+00 -8.493E+00 -16.94

--> element no.   8 :

 1.88E+00  1.808E-07 -3.797E-06  0.000E+00 -2.051E-06  1.049E-06 -4.665E-06
 1.67E-01 -2.603E+00 -8.967E+00 -2.893E+00 -1.641E+00 -2.205E+00 -9.365E+00 -13.64

--> element no.   9 :

 2.12E+00  2.364E-07 -4.175E-06  0.000E+00 -1.678E-06  8.019E-07 -4.740E-06
 1.67E-01 -2.772E+00 -9.830E+00 -3.151E+00 -1.342E+00 -2.526E+00 -1.008E+01 -10.41

--> element no.  10 :

 2.38E+00  2.704E-07 -4.458E-06  0.000E+00 -1.232E-06  5.720E-07 -4.760E-06
 1.67E-01 -2.917E+00 -1.048E+01 -3.350E+00 -9.854E-01 -2.791E+00 -1.061E+01  -7.30

--> element no.  11 :

 2.62E+00  2.876E-07 -4.643E-06  0.000E+00 -7.487E-07  3.988E-07 -4.755E-06
 1.67E-01 -3.024E+00 -1.091E+01 -3.485E+00 -5.990E-01 -2.979E+00 -1.096E+01  -4.32

--> element no.  12 :

 2.88E+00  2.943E-07 -4.735E-06  0.000E+00 -2.508E-07  3.068E-07 -4.747E-06
 1.67E-01 -3.081E+00 -1.113E+01 -3.552E+00 -2.006E-01 -3.076E+00 -1.113E+01  -1.43

--> element no.  13 :

 1.25E-01 -1.141E-06 -1.433E-07  0.000E+00 -1.212E-06  6.682E-07 -1.952E-06
 5.00E-01 -2.853E+00 -1.257E+00 -1.027E+00 -9.693E-01 -7.991E-01 -3.310E+00 -64.73

--> element no.  14 :

 3.75E-01 -9.337E-07 -5.135E-07  0.000E+00 -1.320E-06  6.126E-07 -2.060E-06
```

```
 5.00E-01 -2.652E+00 -1.979E+00 -1.158E+00 -1.056E+00 -1.208E+00 -3.424E+00 -53.83

--> element no.  15 :

 6.25E-01 -7.365E-07 -9.947E-07  0.000E+00 -1.690E-06  8.296E-07 -2.561E-06
 5.00E-01 -2.563E+00 -2.977E+00 -1.385E+00 -1.352E+00 -1.402E+00 -4.138E+00 -40.66

--> element no.  16 :

 8.75E-01 -5.179E-07 -1.547E-06  0.000E+00 -2.067E-06  1.098E-06 -3.163E-06
 5.00E-01 -2.480E+00 -4.127E+00 -1.652E+00 -1.654E+00 -1.456E+00 -5.151E+00 -31.77

--> element no.  17 :

 1.13E+00 -2.583E-07 -2.176E-06  0.000E+00 -2.325E-06  1.298E-06 -3.733E-06
 5.00E-01 -2.361E+00 -5.430E+00 -1.948E+00 -1.860E+00 -1.484E+00 -6.307E+00 -25.24

--> element no.  18 :

 1.38E+00  1.847E-08 -2.844E-06  0.000E+00 -2.392E-06  1.375E-06 -4.201E-06
 5.00E-01 -2.231E+00 -6.811E+00 -2.261E+00 -1.914E+00 -1.537E+00 -7.506E+00 -19.94

--> element no.  19 :

 1.63E+00  2.795E-07 -3.489E-06  0.000E+00 -2.250E-06  1.330E-06 -4.539E-06
 5.00E-01 -2.120E+00 -8.150E+00 -2.568E+00 -1.800E+00 -1.624E+00 -8.646E+00 -15.42

--> element no.  20 :

 1.88E+00  4.843E-07 -4.050E-06  0.000E+00 -1.935E-06  1.198E-06 -4.763E-06
 5.00E-01 -2.077E+00 -9.332E+00 -2.852E+00 -1.548E+00 -1.761E+00 -9.649E+00 -11.56

--> element no.  21 :

 2.13E+00  6.227E-07 -4.491E-06  0.000E+00 -1.523E-06  1.042E-06 -4.910E-06
 5.00E-01 -2.098E+00 -1.028E+01 -3.094E+00 -1.218E+00 -1.921E+00 -1.046E+01  -8.29

--> element no.  22 :

 2.38E+00  6.995E-07 -4.803E-06  0.000E+00 -1.075E-06  9.022E-07 -5.006E-06
 5.00E-01 -2.164E+00 -1.097E+01 -3.283E+00 -8.604E-01 -2.080E+00 -1.105E+01  -5.53

--> element no.  23 :

 2.62E+00  7.351E-07 -4.998E-06  0.000E+00 -6.348E-07  8.045E-07 -5.067E-06
 5.00E-01 -2.234E+00 -1.141E+01 -3.410E+00 -5.078E-01 -2.206E+00 -1.143E+01  -3.16

--> element no.  24 :

 2.88E+00  7.478E-07 -5.090E-06  0.000E+00 -2.093E-07  7.553E-07 -5.098E-06
 5.00E-01 -2.278E+00 -1.162E+01 -3.474E+00 -1.675E-01 -2.275E+00 -1.162E+01  -1.03

--> element no.  25 :
```

```
 1.25E-01 -1.369E-06 -6.731E-08  0.000E+00 -1.406E-06  8.312E-07 -2.268E-06
 8.33E-01 -3.340E+00 -1.257E+00 -1.149E+00 -1.125E+00 -7.656E-01 -3.832E+00 -66.40

--> element no.  26 :

 3.75E-01 -1.261E-06 -2.922E-07  0.000E+00 -1.558E-06  8.545E-07 -2.408E-06
 8.33E-01 -3.261E+00 -1.710E+00 -1.243E+00 -1.246E+00 -1.018E+00 -3.953E+00 -60.95

--> element no.  27 :

 6.25E-01 -1.067E-06 -7.009E-07  0.000E+00 -1.931E-06  1.056E-06 -2.824E-06
 8.33E-01 -3.122E+00 -2.536E+00 -1.414E+00 -1.545E+00 -1.256E+00 -4.401E+00 -50.37

--> element no.  28 :

 8.75E-01 -7.735E-07 -1.293E-06  0.000E+00 -2.402E-06  1.382E-06 -3.449E-06
 8.33E-01 -2.891E+00 -3.723E+00 -1.654E+00 -1.921E+00 -1.341E+00 -5.273E+00 -38.89

--> element no.  29 :

 1.13E+00 -3.882E-07 -2.050E-06  0.000E+00 -2.758E-06  1.662E-06 -4.100E-06
 8.33E-01 -2.572E+00 -5.230E+00 -1.950E+00 -2.207E+00 -1.325E+00 -6.477E+00 -29.47

--> element no.  30 :

 1.38E+00  5.573E-08 -2.890E-06  0.000E+00 -2.809E-06  1.755E-06 -4.589E-06
 8.33E-01 -2.178E+00 -6.892E+00 -2.267E+00 -2.248E+00 -1.278E+00 -7.792E+00 -21.82

--> element no.  31 :

 1.63E+00  4.373E-07 -3.684E-06  0.000E+00 -2.523E-06  1.634E-06 -4.880E-06
 8.33E-01 -1.897E+00 -8.491E+00 -2.597E+00 -2.018E+00 -1.329E+00 -9.060E+00 -15.74

--> element no.  32 :

 1.88E+00  7.145E-07 -4.329E-06  0.000E+00 -2.019E-06  1.423E-06 -5.038E-06
 8.33E-01 -1.748E+00 -9.818E+00 -2.892E+00 -1.615E+00 -1.437E+00 -1.013E+01 -10.91

--> element no.  33 :

 2.13E+00  8.597E-07 -4.787E-06  0.000E+00 -1.450E-06  1.210E-06 -5.137E-06
 8.33E-01 -1.766E+00 -1.080E+01 -3.142E+00 -1.160E+00 -1.620E+00 -1.095E+01  -7.20

--> element no.  34 :

 2.38E+00  9.190E-07 -5.078E-06  0.000E+00 -9.440E-07  1.064E-06 -5.223E-06
 8.33E-01 -1.857E+00 -1.145E+01 -3.327E+00 -7.552E-01 -1.798E+00 -1.151E+01  -4.47

--> element no.  35 :

 2.62E+00  9.357E-07 -5.247E-06  0.000E+00 -5.244E-07  9.798E-07 -5.291E-06
 8.33E-01 -1.952E+00 -1.184E+01 -3.449E+00 -4.195E-01 -1.934E+00 -1.186E+01  -2.42
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--> element no.  36 :

 2.88E+00  9.376E-07 -5.323E-06  0.000E+00 -1.674E-07  9.421E-07 -5.328E-06
 8.33E-01 -2.008E+00 -1.203E+01 -3.508E+00 -1.339E-01 -2.006E+00 -1.203E+01  -0.77

--> element no.  37 :

 1.25E-01 -1.384E-06 -2.681E-08  0.000E+00 -1.458E-06  9.024E-07 -2.314E-06
 1.17E+00 -3.344E+00 -1.172E+00 -1.129E+00 -1.166E+00 -6.644E-01 -3.852E+00 -66.48

--> element no.  38 :

 3.75E-01 -1.348E-06 -1.275E-07  0.000E+00 -1.640E-06  1.012E-06 -2.488E-06
 1.17E+00 -3.338E+00 -1.384E+00 -1.180E+00 -1.312E+00 -7.254E-01 -3.997E+00 -63.33

--> element no.  39 :

 6.25E-01 -1.259E-06 -3.850E-07  0.000E+00 -2.158E-06  1.380E-06 -3.024E-06
 1.17E+00 -3.329E+00 -1.931E+00 -1.315E+00 -1.727E+00 -7.674E-01 -4.493E+00 -56.02

--> element no.  40 :

 8.75E-01 -9.826E-07 -9.667E-07  0.000E+00 -2.982E-06  2.007E-06 -3.956E-06
 1.17E+00 -3.132E+00 -3.106E+00 -1.559E+00 -2.385E+00 -7.335E-01 -5.504E+00 -45.15

--> element no.  41 :

 1.12E+00 -4.215E-07 -1.915E-06  0.000E+00 -3.575E-06  2.484E-06 -4.821E-06
 1.17E+00 -2.544E+00 -4.934E+00 -1.869E+00 -2.860E+00 -6.388E-01 -6.839E+00 -33.66

--> element no.  42 :

 1.37E+00  1.687E-07 -3.034E-06  0.000E+00 -3.572E-06  2.482E-06 -5.347E-06
 1.17E+00 -2.022E+00 -7.147E+00 -2.292E+00 -2.858E+00 -7.462E-01 -8.422E+00 -24.06

--> element no.  43 :

 1.62E+00  6.996E-07 -4.039E-06  0.000E+00 -2.928E-06  2.097E-06 -5.436E-06
 1.17E+00 -1.552E+00 -9.134E+00 -2.672E+00 -2.343E+00 -8.867E-01 -9.799E+00 -15.86

--> element no.  44 :

 1.88E+00  9.061E-07 -4.712E-06  0.000E+00 -1.981E-06  1.534E-06 -5.340E-06
 1.17E+00 -1.595E+00 -1.058E+01 -3.045E+00 -1.584E+00 -1.324E+00 -1.086E+01  -9.71

--> element no.  45 :

 2.12E+00  9.441E-07 -5.079E-06  0.000E+00 -1.227E-06  1.184E-06 -5.319E-06
 1.17E+00 -1.797E+00 -1.143E+01 -3.308E+00 -9.815E-01 -1.698E+00 -1.153E+01  -5.76

--> element no.  46 :
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```
 2.38E+00  9.210E-07 -5.278E-06  0.000E+00 -7.031E-07  9.997E-07 -5.357E-06
 1.17E+00 -2.012E+00 -1.193E+01 -3.486E+00 -5.625E-01 -1.980E+00 -1.196E+01  -3.24

--> element no.  47 :

 2.62E+00  8.882E-07 -5.382E-06  0.000E+00 -3.538E-07  9.081E-07 -5.402E-06
 1.17E+00 -2.174E+00 -1.221E+01 -3.595E+00 -2.830E-01 -2.166E+00 -1.221E+01  -1.61

--> element no.  48 :

 2.88E+00  8.679E-07 -5.426E-06  0.000E+00 -1.068E-07  8.697E-07 -5.428E-06
 1.17E+00 -2.258E+00 -1.233E+01 -3.647E+00 -8.545E-02 -2.257E+00 -1.233E+01  -0.49

--> element no.  49 :

 1.25E-01 -1.282E-06 -1.770E-08  0.000E+00 -1.503E-06  9.805E-07 -2.280E-06
 1.50E+00 -3.091E+00 -1.068E+00 -1.040E+00 -1.202E+00 -5.083E-01 -3.650E+00 -65.04

--> element no.  50 :

 3.75E-01 -1.247E-06 -2.558E-08  0.000E+00 -1.634E-06  1.108E-06 -2.381E-06
 1.50E+00 -3.013E+00 -1.059E+00 -1.018E+00 -1.307E+00 -4.038E-01 -3.668E+00 -63.39

--> element no.  51 :

 6.25E-01 -1.222E-06 -3.044E-08  0.000E+00 -2.145E-06  1.600E-06 -2.852E-06
 1.50E+00 -2.956E+00 -1.050E+00 -1.002E+00 -1.716E+00 -4.031E-02 -3.966E+00 -59.52

--> element no.  52 :

 8.75E-01 -1.074E-06 -4.008E-07  0.000E+00 -3.503E-06  2.781E-06 -4.256E-06
 1.50E+00 -2.899E+00 -1.821E+00 -1.180E+00 -2.802E+00  4.933E-01 -5.214E+00 -50.44

--> element no.  53 :

 1.12E+00 -4.442E-07 -1.721E-06  0.000E+00 -5.079E-06  4.036E-06 -6.201E-06
 1.50E+00 -2.443E+00 -4.485E+00 -1.732E+00 -4.063E+00  7.255E-01 -7.653E+00 -37.95

--> element no.  54 :

 1.37E+00  8.396E-07 -3.580E-06  0.000E+00 -4.673E-06  3.799E-06 -6.539E-06
 1.50E+00 -8.490E-01 -7.920E+00 -2.192E+00 -3.738E+00  7.608E-01 -9.530E+00 -23.30

--> element no.  55 :

 1.62E+00  1.019E-06 -4.740E-06  0.000E+00 -2.658E-06  2.059E-06 -5.780E-06
 1.50E+00 -1.346E+00 -1.056E+01 -2.977E+00 -2.127E+00 -8.791E-01 -1.103E+01 -12.39

--> element no.  56 :

 1.87E+00  8.964E-07 -5.118E-06  0.000E+00 -1.403E-06  1.207E-06 -5.429E-06
 1.50E+00 -1.943E+00 -1.157E+01 -3.378E+00 -1.122E+00 -1.814E+00 -1.170E+01  -6.56
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```
--> element no.  57 :

 2.12E+00  7.513E-07 -5.276E-06  0.000E+00 -6.918E-07  8.297E-07 -5.355E-06
 1.50E+00 -2.418E+00 -1.206E+01 -3.620E+00 -5.535E-01 -2.386E+00 -1.209E+01  -3.27

--> element no.  58 :

 2.38E+00  6.425E-07 -5.343E-06  0.000E+00 -3.148E-07  6.590E-07 -5.359E-06
 1.50E+00 -2.732E+00 -1.231E+01 -3.760E+00 -2.518E-01 -2.725E+00 -1.231E+01  -1.51

--> element no.  59 :

 2.62E+00  5.755E-07 -5.372E-06  0.000E+00 -1.258E-07  5.782E-07 -5.375E-06
 1.50E+00 -2.917E+00 -1.243E+01 -3.837E+00 -1.007E-01 -2.915E+00 -1.243E+01  -0.61

--> element no.  60 :

 2.88E+00  5.443E-07 -5.384E-06  0.000E+00 -3.222E-08  5.445E-07 -5.384E-06
 1.50E+00 -3.001E+00 -1.249E+01 -3.872E+00 -2.578E-02 -3.001E+00 -1.249E+01  -0.16

--> element no.  61 :

 1.25E-01 -1.332E-06  6.098E-08  0.000E+00 -1.574E-06  1.086E-06 -2.356E-06
 1.83E+00 -3.147E+00 -9.191E-01 -1.017E+00 -1.259E+00 -3.520E-01 -3.714E+00 -65.75

--> element no.  62 :

 3.75E-01 -1.324E-06  1.694E-07  0.000E+00 -2.012E-06  1.569E-06 -2.723E-06
 1.83E+00 -3.042E+00 -6.527E-01 -9.237E-01 -1.609E+00  1.570E-01 -3.852E+00 -63.29

--> element no.  63 :

 6.25E-01 -1.109E-06  3.557E-07  0.000E+00 -2.697E-06  2.418E-06 -3.171E-06
 1.83E+00 -2.376E+00 -3.312E-02 -6.023E-01 -2.157E+00  1.250E+00 -3.659E+00 -59.25

--> element no.  64 :

 8.75E-01 -5.890E-07  4.868E-07  0.000E+00 -4.422E-06  4.403E-06 -4.505E-06
 1.83E+00 -1.024E+00  6.970E-01 -8.182E-02 -3.537E+00  3.477E+00 -3.804E+00 -51.84

--> element no.  65 :

 1.12E+00  2.028E-06 -2.346E-06  0.000E+00 -8.293E-06  8.418E-06 -8.735E-06
 1.83E+00  2.992E+00 -4.007E+00 -2.538E-01 -6.634E+00  6.993E+00 -8.008E+00 -31.09

--> element no.  66 :

 1.38E+00  1.286E-06 -5.151E-06  0.000E+00 -2.633E-06  2.226E-06 -6.091E-06
 1.83E+00 -1.034E+00 -1.133E+01 -3.092E+00 -2.107E+00 -6.193E-01 -1.175E+01 -11.12

--> element no.  67 :

 1.62E+00  5.707E-07 -5.174E-06  0.000E+00 -9.611E-07  7.272E-07 -5.330E-06
 1.83E+00 -2.769E+00 -1.196E+01 -3.683E+00 -7.689E-01 -2.706E+00 -1.202E+01  -4.75
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--> element no.  68 :

 1.88E+00  2.469E-07 -5.213E-06  0.000E+00 -3.346E-07  2.674E-07 -5.234E-06
 1.83E+00 -3.578E+00 -1.231E+01 -3.973E+00 -2.677E-01 -3.570E+00 -1.232E+01  -1.75

--> element no.  69 :

 2.12E+00  1.026E-07 -5.212E-06  0.000E+00 -8.743E-08  1.040E-07 -5.213E-06
 1.83E+00 -3.923E+00 -1.243E+01 -4.087E+00 -6.995E-02 -3.923E+00 -1.243E+01  -0.47

--> element no.  70 :

 2.38E+00  4.924E-08 -5.215E-06  0.000E+00  4.120E-09  4.924E-08 -5.215E-06
 1.83E+00 -4.054E+00 -1.248E+01 -4.133E+00  3.296E-03 -4.054E+00 -1.248E+01   0.02

--> element no.  71 :

 2.62E+00  3.520E-08 -5.221E-06  0.000E+00  2.411E-08  3.531E-08 -5.221E-06
 1.83E+00 -4.092E+00 -1.250E+01 -4.148E+00  1.928E-02 -4.092E+00 -1.250E+01   0.13

--> element no.  72 :

 2.88E+00  3.393E-08 -5.224E-06  0.000E+00  1.125E-08  3.396E-08 -5.224E-06
 1.83E+00 -4.098E+00 -1.251E+01 -4.152E+00  9.004E-03 -4.098E+00 -1.251E+01   0.06

--> element no.  73 :

 1.25E-01 -1.634E-06  2.577E-07  0.000E+00 -1.343E-06  9.547E-07 -2.331E-06
 2.12E+00 -3.716E+00 -6.889E-01 -1.101E+00 -1.075E+00 -3.461E-01 -4.058E+00 -72.31

--> element no.  74 :

 3.75E-01 -1.974E-06  8.009E-07  0.000E+00 -2.233E-06  2.042E-06 -3.215E-06
 2.12E+00 -4.097E+00  3.431E-01 -9.384E-01 -1.786E+00  9.726E-01 -4.726E+00 -70.59

--> element no.  75 :

 6.25E-01 -2.228E-06  1.241E-06  0.000E+00 -4.421E-06  4.255E-06 -5.242E-06
 2.12E+00 -4.354E+00  1.195E+00 -7.898E-01 -3.537E+00  2.916E+00 -6.075E+00 -64.06

--> element no.  76 :

 8.75E-01 -3.159E-06  4.116E-06  0.000E+00 -8.317E-06  9.556E-06 -8.600E-06
 2.12E+00 -4.290E+00  7.351E+00  7.654E-01 -6.654E+00  1.037E+01 -7.309E+00 -65.59

--> element no.  77 :

 1.25E-01 -1.931E-06  4.470E-07  0.000E+00 -5.520E-07  5.689E-07 -2.053E-06
 2.38E+00 -4.276E+00 -4.717E-01 -1.187E+00 -4.416E-01 -4.211E-01 -4.327E+00 -83.47

--> element no.  78 :
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 3.75E-01 -2.713E-06  1.436E-06  0.000E+00 -1.011E-06  1.669E-06 -2.946E-06
 2.38E+00 -5.362E+00  1.276E+00 -1.021E+00 -8.086E-01  1.373E+00 -5.459E+00 -83.15

--> element no.  79 :

 6.25E-01 -4.103E-06  2.874E-06  0.000E+00 -2.367E-06  3.601E-06 -4.830E-06
 2.38E+00 -7.547E+00  3.615E+00 -9.830E-01 -1.893E+00  3.928E+00 -7.860E+00 -80.63

--> element no.  80 :

 8.75E-01 -5.936E-06  3.241E-06  0.000E+00 -1.417E-06  3.455E-06 -6.150E-06
 2.38E+00 -1.165E+01  3.031E+00 -2.156E+00 -1.133E+00  3.118E+00 -1.174E+01 -85.61
```

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  OUTPUT   DATA:
  =============
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| node number | Coordinates X | Coordinates Y | Displacements U | Displacements V |
|---|---|---|---|---|
| 1 | 0.000E+00 | 0.000E+00 | -1.916E-47 | -2.129E-47 |
| 2 | 4.163E-16 | 3.333E-01 | -6.345E-27 | -3.225E-27 |
| 3 | 1.490E-15 | 6.667E-01 | -8.446E-27 | -3.780E-27 |
| 4 | 5.829E-16 | 1.000E+00 | -8.959E-27 | -3.952E-27 |
| 5 | -8.882E-16 | 1.333E+00 | -8.526E-27 | -4.010E-27 |
| 6 | 0.000E+00 | 1.667E+00 | -8.050E-27 | -4.229E-27 |
| 7 | 0.000E+00 | 2.000E+00 | -7.563E-27 | -3.558E-27 |
| 8 | 0.000E+00 | 2.250E+00 | -8.732E-27 | -1.759E-27 |
| 9 | 0.000E+00 | 2.500E+00 | -4.625E-27 | 5.940E-48 |
| 10 | 2.500E-01 | 0.000E+00 | -1.979E-27 | -5.565E-27 |
| 11 | 2.500E-01 | 3.333E-01 | -2.423E-07 | -2.229E-07 |
| 12 | 2.500E-01 | 6.667E-01 | -3.281E-07 | -3.185E-07 |
| 13 | 2.500E-01 | 1.000E+00 | -3.566E-07 | -3.633E-07 |
| 14 | 2.500E-01 | 1.333E+00 | -3.357E-07 | -3.812E-07 |
| 15 | 2.500E-01 | 1.667E+00 | -3.053E-07 | -3.930E-07 |
| 16 | 2.500E-01 | 2.000E+00 | -3.606E-07 | -3.523E-07 |
| 17 | 2.500E-01 | 2.250E+00 | -4.564E-07 | -2.235E-07 |
| 18 | 2.500E-01 | 2.500E+00 | -5.089E-07 | -1.254E-27 |
| 19 | 5.000E-01 | 0.000E+00 | -2.701E-27 | -8.260E-27 |
| 20 | 5.000E-01 | 3.333E-01 | -4.177E-07 | -3.693E-07 |
| 21 | 5.000E-01 | 6.667E-01 | -6.196E-07 | -6.161E-07 |
| 22 | 5.000E-01 | 1.000E+00 | -6.958E-07 | -7.660E-07 |
| 23 | 5.000E-01 | 1.333E+00 | -6.705E-07 | -8.332E-07 |
| 24 | 5.000E-01 | 1.667E+00 | -5.938E-07 | -8.384E-07 |
| 25 | 5.000E-01 | 2.000E+00 | -7.341E-07 | -7.661E-07 |
| 26 | 5.000E-01 | 2.250E+00 | -1.070E-06 | -4.945E-07 |
| 27 | 5.000E-01 | 2.500E+00 | -1.252E-06 | -6.561E-27 |
| 28 | 7.500E-01 | 0.000E+00 | -3.388E-27 | -1.077E-26 |
| 29 | 7.500E-01 | 3.333E-01 | -5.513E-07 | -5.178E-07 |
| 30 | 7.500E-01 | 6.667E-01 | -8.543E-07 | -9.341E-07 |

| | | | | |
|---|---|---|---|---|
| 31 | 7.500E-01 | 1.000E+00 | -9.946E-07 | -1.251E-06 |
| 32 | 7.500E-01 | 1.333E+00 | -1.001E-06 | -1.441E-06 |
| 33 | 7.500E-01 | 1.667E+00 | -8.740E-07 | -1.456E-06 |
| 34 | 7.500E-01 | 2.000E+00 | -1.008E-06 | -1.291E-06 |
| 35 | 7.500E-01 | 2.250E+00 | -1.910E-06 | -9.425E-07 |
| 36 | 7.500E-01 | 2.500E+00 | -2.463E-06 | -8.678E-27 |
| 37 | 1.000E+00 | 0.000E+00 | -3.881E-27 | -1.345E-26 |
| 38 | 1.000E+00 | 3.333E-01 | -6.438E-07 | -6.802E-07 |
| 39 | 1.000E+00 | 6.667E-01 | -1.021E-06 | -1.295E-06 |
| 40 | 1.000E+00 | 1.000E+00 | -1.215E-06 | -1.840E-06 |
| 41 | 1.000E+00 | 1.333E+00 | -1.272E-06 | -2.295E-06 |
| 42 | 1.000E+00 | 1.667E+00 | -1.140E-06 | -2.547E-06 |
| 43 | 1.000E+00 | 2.000E+00 | -1.037E-06 | -2.388E-06 |
| 44 | 1.000E+00 | 2.250E+00 | -3.461E-06 | -6.783E-07 |
| 45 | 1.000E+00 | 2.500E+00 | -3.880E-06 | -1.563E-27 |
| 46 | 1.250E+00 | 0.000E+00 | -4.124E-27 | -1.628E-26 |
| 47 | 1.250E+00 | 3.333E-01 | -6.899E-07 | -8.546E-07 |
| 48 | 1.250E+00 | 6.667E-01 | -1.104E-06 | -1.690E-06 |
| 49 | 1.250E+00 | 1.000E+00 | -1.326E-06 | -2.512E-06 |
| 50 | 1.250E+00 | 1.333E+00 | -1.372E-06 | -3.334E-06 |
| 51 | 1.250E+00 | 1.667E+00 | -1.263E-06 | -4.229E-06 |
| 52 | 1.250E+00 | 2.000E+00 | 1.001E-07 | -5.952E-06 |
| 55 | 1.500E+00 | 0.000E+00 | -4.088E-27 | -1.912E-26 |
| 56 | 1.500E+00 | 3.333E-01 | -6.859E-07 | -1.031E-06 |
| 57 | 1.500E+00 | 6.667E-01 | -1.098E-06 | -2.091E-06 |
| 58 | 1.500E+00 | 1.000E+00 | -1.303E-06 | -3.196E-06 |
| 59 | 1.500E+00 | 1.333E+00 | -1.310E-06 | -4.397E-06 |
| 60 | 1.500E+00 | 1.667E+00 | -9.046E-07 | -5.888E-06 |
| 61 | 1.500E+00 | 2.000E+00 | 3.853E-07 | -7.599E-06 |
| 64 | 1.750E+00 | 0.000E+00 | -3.776E-27 | -2.180E-26 |
| 65 | 1.750E+00 | 3.333E-01 | -6.348E-07 | -1.195E-06 |
| 66 | 1.750E+00 | 6.667E-01 | -1.010E-06 | -2.461E-06 |
| 67 | 1.750E+00 | 1.000E+00 | -1.173E-06 | -3.811E-06 |
| 68 | 1.750E+00 | 1.333E+00 | -1.090E-06 | -5.303E-06 |
| 69 | 1.750E+00 | 1.667E+00 | -6.148E-07 | -6.972E-06 |
| 70 | 1.750E+00 | 2.000E+00 | 3.809E-07 | -8.710E-06 |
| 73 | 2.000E+00 | 0.000E+00 | -3.236E-27 | -2.414E-26 |
| 74 | 2.000E+00 | 3.333E-01 | -5.444E-07 | -1.336E-06 |
| 75 | 2.000E+00 | 6.667E-01 | -8.581E-07 | -2.770E-06 |
| 76 | 2.000E+00 | 1.000E+00 | -9.679E-07 | -4.306E-06 |
| 77 | 2.000E+00 | 1.333E+00 | -8.427E-07 | -5.955E-06 |
| 78 | 2.000E+00 | 1.667E+00 | -4.142E-07 | -7.699E-06 |
| 79 | 2.000E+00 | 2.000E+00 | 3.037E-07 | -9.436E-06 |
| 82 | 2.250E+00 | 0.000E+00 | -2.530E-27 | -2.603E-26 |
| 83 | 2.250E+00 | 3.333E-01 | -4.262E-07 | -1.447E-06 |
| 84 | 2.250E+00 | 6.667E-01 | -6.650E-07 | -3.006E-06 |
| 85 | 2.250E+00 | 1.000E+00 | -7.312E-07 | -4.662E-06 |
| 86 | 2.250E+00 | 1.333E+00 | -6.073E-07 | -6.399E-06 |
| 87 | 2.250E+00 | 1.667E+00 | -2.738E-07 | -8.173E-06 |
| 88 | 2.250E+00 | 2.000E+00 | 2.147E-07 | -9.910E-06 |
| 91 | 2.500E+00 | 0.000E+00 | -1.726E-27 | -2.740E-26 |
| 92 | 2.500E+00 | 3.333E-01 | -2.910E-07 | -1.525E-06 |
| 93 | 2.500E+00 | 6.667E-01 | -4.504E-07 | -3.168E-06 |

| | | | | |
|---|---|---|---|---|
| 94 | 2.500E+00 | 1.000E+00 | -4.862E-07 | -4.897E-06 |
| 95 | 2.500E+00 | 1.333E+00 | -3.918E-07 | -6.679E-06 |
| 96 | 2.500E+00 | 1.667E+00 | -1.681E-07 | -8.467E-06 |
| 97 | 2.500E+00 | 2.000E+00 | 1.336E-07 | -1.021E-05 |
| 100 | 2.750E+00 | 0.000E+00 | -8.722E-28 | -2.822E-26 |
| 101 | 2.750E+00 | 3.333E-01 | -1.472E-07 | -1.571E-06 |
| 102 | 2.750E+00 | 6.667E-01 | -2.267E-07 | -3.260E-06 |
| 103 | 2.750E+00 | 1.000E+00 | -2.421E-07 | -5.028E-06 |
| 104 | 2.750E+00 | 1.333E+00 | -1.918E-07 | -6.834E-06 |
| 105 | 2.750E+00 | 1.667E+00 | -8.035E-08 | -8.627E-06 |
| 106 | 2.750E+00 | 2.000E+00 | 6.338E-08 | -1.037E-05 |
| 109 | 3.000E+00 | 0.000E+00 | 4.224E-47 | -1.424E-26 |
| 110 | 3.000E+00 | 3.333E-01 | 6.915E-27 | -1.586E-06 |
| 111 | 3.000E+00 | 6.667E-01 | 5.512E-27 | -3.290E-06 |
| 112 | 3.000E+00 | 1.000E+00 | 5.497E-27 | -5.070E-06 |
| 113 | 3.000E+00 | 1.333E+00 | 6.826E-27 | -6.883E-06 |
| 114 | 3.000E+00 | 1.667E+00 | 9.294E-27 | -8.678E-06 |
| 115 | 3.000E+00 | 2.000E+00 | 5.742E-27 | -1.042E-05 |

>>> END-OF-FILE encountered during read <<<