

T1DHC

A Finite Element Computer Program for One-Dimensional Transient Heat Conduction Analyses

written by

V. N. Kaliakin
Department of Civil Engineering
University of Delaware

I. INTRODUCTION

The computer program T1DHC described herein was developed to evaluate finite element analyses of heat conduction in a one-dimensional continuum under time dependent (transient) conditions. T1DHC is intended for instructional use. It is thus liberally commented and uses descriptive variable names.

II. ALGORITHMS USED

The approximate solution to the one-dimensional transient heat conduction problem is obtained using algorithms which are members of the so-called *generalized trapezoidal family*. The general form of these algorithms is

$$([\mathbf{C}] + \theta \Delta t [\mathbf{K}]) \{\hat{\mathbf{T}}^{n+1}\} = ([\mathbf{C}] + (\theta - 1) \Delta t [\mathbf{K}]) \{\hat{\mathbf{T}}^n\} + \Delta t \{\mathbf{F}\}$$

where

$[\mathbf{C}]$ = system capacitance matrix. In the current version of T1DHC, either a consistent or a “lumped” formulation may be used to compute the element capacitance matrices which are assembled to form $[\mathbf{C}]$.

$[\mathbf{K}]$ = system conductivity matrix.

$\{\mathbf{F}\}$ = system “source” vector. In the current version of T1DHC this vector is assumed to be *independent* of time.

Δt = solution time step. In the current version of TIDHC the time step is assumed to be *constant* throughout the solution.

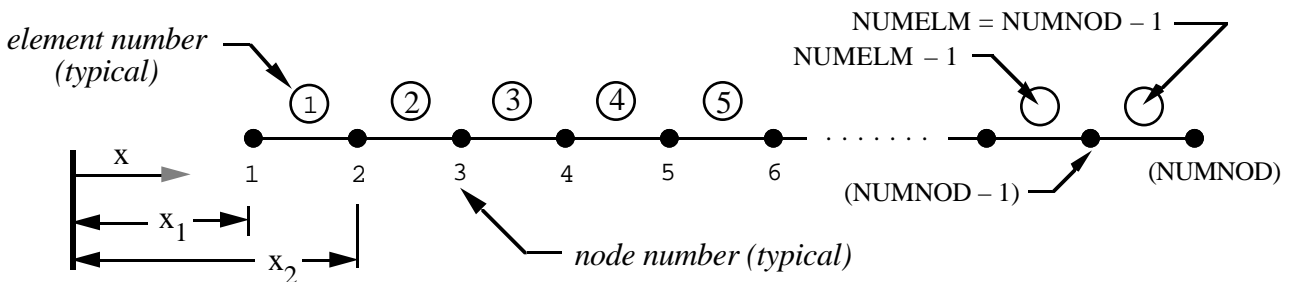
$\{\hat{T}^n\}$ = vector of approximate (known) nodal temperatures at the previous time step.

$\{\hat{T}^{n+1}\}$ = vector of (unknown) approximate nodal temperatures at the present time step. At each step in the solution, these values represent the unknowns being solved for.

θ = parameter ($0 \leq \theta \leq 1$) defining a particular member of the generalized trapezoidal family of algorithms.

III. ELEMENT TOPOLOGY AND GLOBAL COORDINATE SYSTEM

The finite element analysis utilizes a linear approximation for the temperature within each element. As such, the discretization of the solution domain is achieved using two-noded elements. The nodes and elements should be sequentially numbered in the following manner:



IV. UNITS

The units used for various input quantities must be consistent, and determine the units of the output. In the subsequent description of the program input, the units associated with input quantities are described using the following conventions:

E : denotes units of energy, heat, or work (e.g., Joules, calories, Btu, etc.)

L : denotes units of length (e.g., meters, centimeters, inches, etc.)

M : denotes units of mass (e.g., kilograms, grams, slugs, etc.)

t : denotes units of time (e.g., seconds, minutes, etc.)

T : denotes units of temperature (e.g., degrees Kelvin, degrees Celsius, etc.)

V. PROGRAM INPUT

Data input to T1DHC is quite simple. With the exception of alphanumeric strings, all input is performed with list-directed READ statements (i.e., free-format). Since input is terminated by exhaustion of the input list rather than by line boundaries, list-directed input records may span several lines. Multiple spaces between input fields are ignored and “null” fields in the input record may leave the value of a particular variable unchanged (as opposed to setting it to zero). The safest way to avoid erroneous input is to include explicit values for all input variables that are being read; this ensures that the correct mathematical model will be analyzed. When alphanumeric strings are required in the input, each of them should appear on a separate line and *must not be preceded by a blank record*. Users not familiar with list-directed input are advised to consult appropriate documentation for their computer.

In the subsequent description of the input quantities required by the program the variable types (i.e., alphanumeric, integer, or real) are given in brackets []. Numbers in braces { } signify that a note explaining the particular input quantity is provided in the following section. Since the safest way to avoid erroneous list-directed input is to include explicit values for all variables that are being read, no default values are set within the program. Note also that only very rudimentary error checking is performed within the T1DHC computer program.

The data required by the program consists of the following quantities:

1. Identification Title Record (A72)

TITLE [alphanumeric] : any title describing the analysis.

2. Control Record

NUMNOD [integer] = total number of node points used in the analysis (2).

NUMMAT [integer] = total number of material types used in modeling the body (1).

NUMLOD [integer] = total number of different load sets used in the analysis (0).

NUMSPE [integer] = total number of records containing nodal specifications (1).

3. Parameters Associated with the Time Dependent Solution

KIND [integer] = $\left\{ \begin{array}{l} 1 : \text{a consistent formulation is used to evaluate the element} \\ \text{capacitance matrices} \\ 2 : \text{a lumped formulation is used to evaluate the element} \\ \text{capacitance matrices} \end{array} \right.$

THETA [real] = parameter defining the particular finite difference scheme {1}
used to approximate the time derivative of temperature.

NSTEPS [integer] = number of steps used in the solution.

DT [real] = constant time increment Δt used for each solution step.

4. Output Control Record {2}

ISTART [integer] = number of the first solution step for which output is desired (1).

IEND [integer] = number of the last solution step for which output is desired.

INCR [integer] = numbering increment for the sequence of solution steps specified.

5. Material Data

Supply NUMMAT records to describe all the different material types found in the mathematical model. These records may be supplied in *any order*.

NUMBER [integer] = material number

CONDUCT [real] = value of the thermal conductivity (units : $E t^{-1} L^{-1} T^{-1}$).
For a given element, the thermal conductivity is assumed *constant*.

DENSTY [real] = value of the mass density (units : ML^{-3}).
For a given element, the mass density is assumed *constant*.

SPHEAT [real] = value of the specific heat (units : $E M^{-1} T^{-1}$).
For a given element, the specific heat is assumed *constant*.

6. Thermal Load Set Data

Supply NUMLOD records to describe all the different thermal load sets applied to the mathematical model (if NUMLOD = 0, skip this portion of the input entirely). These records may be supplied in *any order*.

NUMBER [integer] = load set number {3}

SOURC1 [real] = value of the source term S_1 (units : $\text{Et}^{-1}\text{L}^{-3}$).
Over a given element, the source term is assumed *constant*.

SOURC2 [real] = value of the source term S_2 (units : $\text{Et}^{-1}\text{L}^{-3}\text{T}^{-1}$).
Over a given element, the source term is assumed *constant*.

7. Node Point Data

Supply NUMNOD records to describe all the node points in the mathematical model. These records may be supplied in *any order*.

NUMBER [integer] = node point number {4}

XY [real] = x coordinate of the node point (units : L)

NUMEND [integer] = number of the final node point to be generated¹.
If node point generation is *not* desired, set NUMEND equal to NUMBER.

XYEND [real] = x- or y coordinate of the final node point to be generated¹.

INCR [integer] = numbering increment for successive node points¹.

RATIO [real] = spacing ratio for successive node points¹.

TINIT [real] = initial value of temperature at the node point (units : T)

¹ this quantity is associated with the straight line generation of node point locations; further details are given in program note {4}.

8. Element Information

Supply (NUMNOD – 1) records to describe all the elements in the mathematical model. These records may be supplied in *any order*.

NUMBER [integer] = element number

NODE1 [integer] = number of the node point at one end of the element {5}

NODE2 [integer] = number of the node point at other end of the element

MATNUM [integer] = number of the material associated with the element

LODNUM [integer] = number of the load set associated with the element.
If NUMLOD = 0, set LODNUM equal to any integer 1.

AREA [real] = cross-sectional area of the element.
Over a given element, the source term is assumed *constant*.

NUMADD [integer] = number of additional elements to be generated².
If element generation is *not* desired, set NUMADD equal to zero.

INCR [integer] = numbering increment for corresponding nodes in successive elements².

9. Nodal Specification Data

Supply NUMSPE records to describe all the records containing nodal specifications. These records may be supplied in *any order*.

NUMBER [integer] = number of node point at which the specification is to be applied.

IFLAG [integer] = $\left\{ \begin{array}{l} 0 : \text{if a known value of concentrated heat flux} \\ \text{is applied at node NUMBER} \\ \\ 1 : \text{if a known value of temperature} \\ \text{is specified at node NUMBER} \end{array} \right. \begin{array}{l} \{6\} \\ \\ \{7\} \end{array}$

VALUE [real] = specified value of $\left\{ \begin{array}{l} \text{heat flux per unit area (units : E t}^{-1}\text{L}^{-2}\text{)} \\ \\ \text{temperature (units : T)} \end{array} \right.$

² this quantity is associated with the generation of element information; further details are given in program note {5}.

This completes the description of input quantities. To include several analyses in a single data file and to subsequently solve them as “stacked jobs”, repeat the above sequence of records for each analysis. *NOTE*: blank lines cannot precede title records in the data file.

VI. NOTES REGARDING PROGRAM INPUT

{1} The value selected for **THETA** defines the particular finite difference scheme used to approximate the time derivative of temperature. As such, the value of **THETA** determines which member of the generalized trapezoidal family of algorithms is to be used. More precisely

$\theta = 0$: Forward difference ; Forward Euler method.

$\theta = \frac{1}{2}$: Central Difference; Midpoint Rule ; Crank-Nicolson method.

$\theta = \frac{2}{3}$: “Galerkin” scheme.

$\theta = 1$: Backward difference ; Backward Euler method.

- For $\frac{1}{2} < \theta < 1$ the methods are *unconditionally stable*, though numerical oscillations will occur at large values of Δt . For $\theta = 1$ (backward difference) the method is unconditionally stable and does *not* oscillate.
- With the exception of the Central Difference method (which is *second-order* accurate), the members of the generalized trapezoidal family of algorithms are *first-order* accurate.

{2} By their nature, time marching schemes have the capacity for generating large amounts of output data (i.e., the results). The output control record is used to regulate the frequency with which the results are printed. The results associated with the first and last solution steps are *always printed*. The values chosen for **ISTART**, **IEND** and **INCR** are thus used to specify the frequency of output for the remaining time steps. For example, to have the solution printed for every third step beginning with the fourth and ending with the twenty-eighth, specify

4 28 3

for the output control record.

{3} The total source term has the following general form:

$$S(x) = S_1 + S_2 T$$

where T represents the temperature.

{4} To assist in defining the the locations of node points, a node generation scheme has been incorporated into TIDHC. When generation of a sequence of node points is desired it is only necessary to input the following information (on a single record):

- the numbers of the initial and final node points in the sequence (in the Program Input these points are denoted by **NUMBER** and **NUMEND**, respectively);
- the coordinated of the initial and final node points in the sequence (denoted in the Program Input by **XY** and **XYEND**, respectively);
- the value of **INCR**, which represents the difference between any two successive node numbers in the sequence; and,
- the value of **RATIO**, which defines the ratio of the distance between any two adjacent pairs of nodes. A value of **RATIO** = 1.0 results in *equally spaced* node points.
- the value of **TINIT** represents the initial temperature at each node in the sequence. NOTE: as currently coded, the initial temperature at each node point in the sequence must be the *same*.

In selecting a value of **RATIO**, the following three formulas are of use (in the sequel denote the number of segments comprising a sequence of nodes by “m”):

- 1) If the total length of a line of nodes is equal to L and if **RATIO** = 1.0, then the length of the first segment will equal

$$\frac{L (1 - \mathbf{RATIO})}{(1 - \mathbf{RATIO}^m)}$$

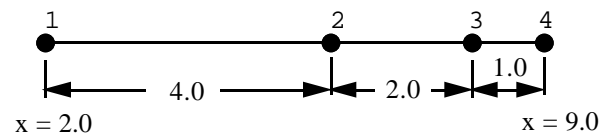
- 2) If the ratio of the lengths of the last and the first segments must equal a specified value “D” (i.e., $D = \Delta L_m / \Delta L_1$), then specify a value of **RATIO** equal to D^r , where $r = (m - 1)^{-1}$ and where $m > 1$.

3) Suppose a line of nodes has been generated with a particular value of **RATIO** and that this line is to now be regenerated with *twice* as many segments (i.e., with twice as many node points minus one). Furthermore suppose that it is desired to have the locations of the first set of nodes be retained in the second set. To achieve the above-mentioned goals use $\sqrt{\mathbf{RATIO}}$ as the new spacing ratio.

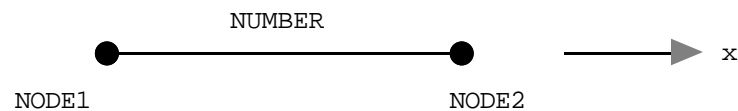
In generating a sequence of nodes, the end points may be entered in *either* order. For example, the sequence illustrated below can be defined by specifying either

1 2.0 4 9.0 1 0.5 or 4 9.0 1 2.0 -1 2.0

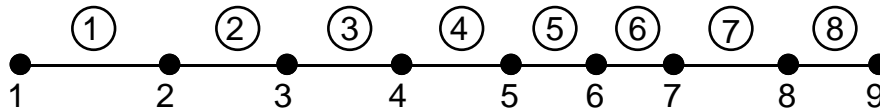
The spacing of the intermediate points (nodes 2 and 3) is controlled by the value selected for **RATIO**.



{5} Element number “NUMBER” has the following configuration:



If the material number (**MATNUM**), the load set number (**LODNUM**) and the cross-sectional area (**AREA**) are the same for a contiguous sequence of elements, the node numbers associated with these elements can be simply established by means of the element generation option available in T1DHC. To generate a sequence of elements, node points (**NODE1** and **NODE2**) are specified only for the first element, together with values for **MATNUM**, **LODNUM**, **AREA**, **NUMADD** and **INCR**. For example, in the grid shown below, all the elements possess identical material numbers (equal to 2) and cross-sectional areas (equal to 24.0) and are subjected to a loading described by load set 1.



The record required to specify all information for element 1, and to generate all information for elements 2 through 8 is thus:

1 1 2 2 1 24.0 7 1

- {6} Concentrated (nodal) values of heat flux (input or output) may be specified at any number of locations along the body by placing node points at these locations. For each such node point **IFLAG** is set equal to zero (0) and **VALUE** is set equal to the known value of heat flux. A positive heat flux indicates that heat is being input to the body.
- {7} Known values of temperature may be specified at any number of locations along the body by placing node points at these locations. For each such node point **IFLAG** is set equal to one (1) and **VALUE** is set equal to the known value of temperature.

NOTE: At node points where the temperature is unknown and there is no exchange of heat with the surroundings, **IFLAG** should be set equal to zero (0) and **VALUE** should be set equal to 0.0. Since these values are pre-set upon initiation of the program, *there is no need to supply nodal specification records for such cases.*

VII. PROGRAM OUTPUT

The output generated by T1DHC consists of:

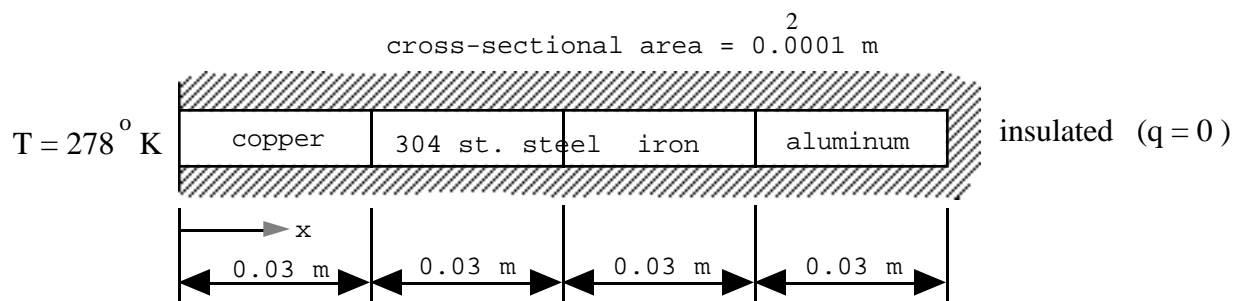
- an “echo” of the input data, including the node point- and element data input and/or generated;
- the initial values of the heat flux at the element centers (the coordinates of these points are also printed);
- then for each of the solution steps specified in the Output Control Record, as well as the first and last solution steps, the following information is printed:

-> the approximate values of the temperature at the nodes. Intermediate values can of course be found by returning to the linear approximation for temperature within each element; and,

-> the approximate values of the heat flux at the element centers (the coordinates of these points are also printed).

VIII. SAMPLE ANALYSIS

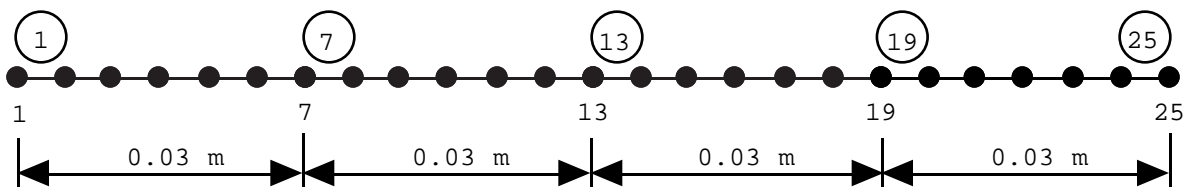
One-dimensional transient heat conduction : Consider the composite rod shown below.



For $0 < x < 0.12$, the initial (uniform) distribution of temperature is equal to 320° K . The properties of the above materials are:

Material	ρ (kg/m ³)	c (J/kg-°K)	k (W/m-°K)
Copper	8933	385.0	400.0
304 Stainless Steel	7900	477.0	21.00
Iron	7870	440.0	53.00
Aluminum	2702	902.0	228.0

The following finite element grid is used to model the above rod:



A consistent form of the element capacitance matrices is used in conjunction with the Central Difference Method (i.e., $\theta = 0.5$). One-hundred solution time steps are used with $\Delta t = 1.0$ second. Output is requested every twentieth solution step beginning with the twentieth and ending with the one-hundredth (the results at the first and at the last solution step are, of course, always printed). A copy of the complete input data file for this analysis is shown below.

sample analysis using the T1DHC computer program

```

25  4  0  2

1  0.50  100  1.0

20  100  20

1  400.0  8933.0  385.0
4  228.0  2702.0  902.0
3   53.0  7870.0  440.0
2   21.0  7900.0  477.0

1  0.0  25  0.12  1  1.0  320.0

1  1  2  1  1  0.0001  5  1
7  7  8  2  1  0.0001  5  1
13 13 14 3  1  0.0001  5  1
19 19 20 4  1  0.0001  5  1

1  1  278.0
25 0  0.0

```

The results obtained using the above data in conjunction with the T1DHC computer program are shown below.

sample analysis using the T1DHC computer program

Input data for one-dimensional steady transient conduction analysis:

Total number of NODES = 25
 Total number of ELEMENTS = 24
 Total number of MATERIALS = 4
 Total number of LOAD SETS = 0
 Total number of NODAL SPECIFICATIONS = 2

CONSISTENT formulation employed
 Value of solution parameter THETA = 0.50
 Total number of solution TIME STEPS = 100
 Time increment = 1.000E+00

MATERIAL DATA:

material number	thermal conductivity	mass density	specific heat
-----	-----	-----	-----
1	4.000E+02	8.933E+03	3.850E+02
2	2.100E+01	7.900E+03	4.770E+02
3	5.300E+01	7.870E+03	4.400E+02
4	2.280E+02	2.702E+03	9.020E+02

NODE POINT DATA:

node number	nodal coordinate	initial temperature
-----	-----	-----
1	0.000E+00	3.200E+02
2	5.000E-03	3.200E+02
3	1.000E-02	3.200E+02
4	1.500E-02	3.200E+02
5	2.000E-02	3.200E+02
6	2.500E-02	3.200E+02
7	3.000E-02	3.200E+02
8	3.500E-02	3.200E+02
9	4.000E-02	3.200E+02
10	4.500E-02	3.200E+02
11	5.000E-02	3.200E+02
12	5.500E-02	3.200E+02
13	6.000E-02	3.200E+02
14	6.500E-02	3.200E+02

15	7.000E-02	3.200E+02
16	7.500E-02	3.200E+02
17	8.000E-02	3.200E+02
18	8.500E-02	3.200E+02
19	9.000E-02	3.200E+02
20	9.500E-02	3.200E+02
21	1.000E-01	3.200E+02
22	1.050E-01	3.200E+02
23	1.100E-01	3.200E+02
24	1.150E-01	3.200E+02
25	1.200E-01	3.200E+02

ELEMENT DATA:

Element number	node		material number	load set number	area
-----	1	2	-----	-----	-----
1	1	2	1	1	1.000E-04
2	2	3	1	1	1.000E-04
3	3	4	1	1	1.000E-04
4	4	5	1	1	1.000E-04
5	5	6	1	1	1.000E-04
6	6	7	1	1	1.000E-04
7	7	8	2	1	1.000E-04
8	8	9	2	1	1.000E-04
9	9	10	2	1	1.000E-04
10	10	11	2	1	1.000E-04
11	11	12	2	1	1.000E-04
12	12	13	2	1	1.000E-04
13	13	14	3	1	1.000E-04
14	14	15	3	1	1.000E-04
15	15	16	3	1	1.000E-04
16	16	17	3	1	1.000E-04
17	17	18	3	1	1.000E-04
18	18	19	3	1	1.000E-04
19	19	20	4	1	1.000E-04
20	20	21	4	1	1.000E-04
21	21	22	4	1	1.000E-04
22	22	23	4	1	1.000E-04
23	23	24	4	1	1.000E-04
24	24	25	4	1	1.000E-04

NODAL SPECIFICATIONS:

node number	specification
-----	-----

1 specified temperature = 2.780E+02
 25 conc. heat flux/area = 0.000E+00

Initial conditions on element heat flux:

```
-----
```

element		heat flux	heat flux per unit area
number	center		
-----	-----	-----	-----
1	2.500E-03	0.000E+00	0.000E+00
2	7.500E-03	0.000E+00	0.000E+00
3	1.250E-02	0.000E+00	0.000E+00
4	1.750E-02	0.000E+00	0.000E+00
5	2.250E-02	0.000E+00	0.000E+00
6	2.750E-02	0.000E+00	0.000E+00
7	3.250E-02	0.000E+00	0.000E+00
8	3.750E-02	0.000E+00	0.000E+00
9	4.250E-02	0.000E+00	0.000E+00
10	4.750E-02	0.000E+00	0.000E+00
11	5.250E-02	0.000E+00	0.000E+00
12	5.750E-02	0.000E+00	0.000E+00
13	6.250E-02	0.000E+00	0.000E+00
14	6.750E-02	0.000E+00	0.000E+00
15	7.250E-02	0.000E+00	0.000E+00
16	7.750E-02	0.000E+00	0.000E+00
17	8.250E-02	0.000E+00	0.000E+00
18	8.750E-02	0.000E+00	0.000E+00
19	9.250E-02	0.000E+00	0.000E+00
20	9.750E-02	0.000E+00	0.000E+00
21	1.025E-01	0.000E+00	0.000E+00
22	1.075E-01	0.000E+00	0.000E+00
23	1.125E-01	0.000E+00	0.000E+00
24	1.175E-01	0.000E+00	0.000E+00

Results for time T = 1.000E+00:

```
-----
```

node	temperature
number	-----
1	2.780E+02
2	2.985E+02
3	3.089E+02
4	3.143E+02
5	3.170E+02

6	3.183E+02
7	3.188E+02
8	3.201E+02
9	3.200E+02
10	3.200E+02
11	3.200E+02
12	3.200E+02
13	3.200E+02
14	3.200E+02
15	3.200E+02
16	3.200E+02
17	3.200E+02
18	3.200E+02
19	3.200E+02
20	3.200E+02
21	3.200E+02
22	3.200E+02
23	3.200E+02
24	3.200E+02
25	3.200E+02

element			
number	center	heat flux	heat flux per unit area
-----	-----	-----	-----
1	2.500E-03	-1.636E+02	-1.636E+06
2	7.500E-03	-8.382E+01	-8.382E+05
3	1.250E-02	-4.282E+01	-4.282E+05
4	1.750E-02	-2.165E+01	-2.165E+05
5	2.250E-02	-1.051E+01	-1.051E+05
6	2.750E-02	-4.239E+00	-4.239E+04
7	3.250E-02	-5.191E-01	-5.191E+03
8	3.750E-02	3.235E-02	3.235E+02
9	4.250E-02	-2.038E-03	-2.038E+01
10	4.750E-02	1.410E-04	1.410E+00
11	5.250E-02	-1.282E-05	-1.282E-01
12	5.750E-02	1.282E-05	1.282E-01
13	6.250E-02	0.000E+00	0.000E+00
14	6.750E-02	3.235E-05	3.235E-01
15	7.250E-02	0.000E+00	0.000E+00
16	7.750E-02	0.000E+00	0.000E+00
17	8.250E-02	0.000E+00	0.000E+00
18	8.750E-02	-6.470E-05	-6.470E-01
19	9.250E-02	1.392E-04	1.392E+00
20	9.750E-02	2.783E-04	2.783E+00
21	1.025E-01	0.000E+00	0.000E+00
22	1.075E-01	0.000E+00	0.000E+00
23	1.125E-01	0.000E+00	0.000E+00
24	1.175E-01	0.000E+00	0.000E+00

Results for time T = 2.000E+01:

```

-----
node
number          temperature
-----
1              2.780E+02
2              2.787E+02
3              2.796E+02
4              2.803E+02
5              2.811E+02
6              2.818E+02
7              2.824E+02
8              2.943E+02
9              3.040E+02
10             3.111E+02
11             3.156E+02
12             3.181E+02
13             3.194E+02
14             3.197E+02
15             3.198E+02
16             3.199E+02
17             3.200E+02
18             3.200E+02
19             3.200E+02
20             3.200E+02
21             3.200E+02
22             3.200E+02
23             3.200E+02
24             3.200E+02
25             3.200E+02

```

```

element
number  center      heat flux      heat flux per unit area
-----  -----
1       2.500E-03     -5.801E+00     -5.801E+04
2       7.500E-03     -7.153E+00     -7.153E+04
3       1.250E-02     -5.278E+00     -5.278E+04
4       1.750E-02     -6.193E+00     -6.193E+04
5       2.250E-02     -5.667E+00     -5.667E+04
6       2.750E-02     -5.340E+00     -5.340E+04
7       3.250E-02     -4.975E+00     -4.975E+04
8       3.750E-02     -4.098E+00     -4.098E+04
9       4.250E-02     -2.966E+00     -2.966E+04
10      4.750E-02     -1.887E+00     -1.887E+04
11      5.250E-02     -1.060E+00     -1.060E+04
12      5.750E-02     -5.461E-01     -5.461E+03
13      6.250E-02     -2.975E-01     -2.975E+03
14      6.750E-02     -1.652E-01     -1.652E+03
15      7.250E-02     -8.673E-02     -8.673E+02
16      7.750E-02     -4.296E-02     -4.296E+02
17      8.250E-02     -2.022E-02     -2.022E+02

```

18	8.750E-02	-9.640E-03	-9.640E+01
19	9.250E-02	-5.566E-03	-5.566E+01
20	9.750E-02	-3.618E-03	-3.618E+01
21	1.025E-01	-2.505E-03	-2.505E+01
22	1.075E-01	-1.809E-03	-1.809E+01
23	1.125E-01	-8.350E-04	-8.350E+00
24	1.175E-01	-2.783E-04	-2.783E+00

Results for time T = 4.000E+01:

node
number temperature

1	2.780E+02
2	2.785E+02
3	2.789E+02
4	2.794E+02
5	2.799E+02
6	2.803E+02
7	2.807E+02
8	2.889E+02
9	2.965E+02
10	3.031E+02
11	3.086E+02
12	3.129E+02
13	3.162E+02
14	3.172E+02
15	3.180E+02
16	3.186E+02
17	3.191E+02
18	3.194E+02
19	3.196E+02
20	3.197E+02
21	3.197E+02
22	3.198E+02
23	3.198E+02
24	3.198E+02
25	3.198E+02

 element
number center heat flux heat flux per unit area

1	2.500E-03	-3.689E+00	-3.689E+04
2	7.500E-03	-3.822E+00	-3.822E+04
3	1.250E-02	-3.576E+00	-3.576E+04
4	1.750E-02	-3.746E+00	-3.746E+04
5	2.250E-02	-3.562E+00	-3.562E+04
6	2.750E-02	-3.565E+00	-3.565E+04

7	3.250E-02	-3.444E+00	-3.444E+04
8	3.750E-02	-3.178E+00	-3.178E+04
9	4.250E-02	-2.770E+00	-2.770E+04
10	4.750E-02	-2.290E+00	-2.290E+04
11	5.250E-02	-1.813E+00	-1.813E+04
12	5.750E-02	-1.399E+00	-1.399E+04
13	6.250E-02	-1.086E+00	-1.086E+04
14	6.750E-02	-8.400E-01	-8.400E+03
15	7.250E-02	-6.371E-01	-6.371E+03
16	7.750E-02	-4.755E-01	-4.755E+03
17	8.250E-02	-3.512E-01	-3.512E+03
18	8.750E-02	-2.594E-01	-2.594E+03
19	9.250E-02	-2.008E-01	-2.008E+03
20	9.750E-02	-1.595E-01	-1.595E+03
21	1.025E-01	-1.209E-01	-1.209E+03
22	1.075E-01	-8.461E-02	-8.461E+02
23	1.125E-01	-5.024E-02	-5.024E+02
24	1.175E-01	-1.656E-02	-1.656E+02

Results for time T = 6.000E+01:

```

-----
node
number          temperature
-----
1              2.780E+02
2              2.784E+02
3              2.787E+02
4              2.791E+02
5              2.795E+02
6              2.798E+02
7              2.802E+02
8              2.869E+02
9              2.933E+02
10             2.993E+02
11             3.046E+02
12             3.092E+02
13             3.132E+02
14             3.145E+02
15             3.157E+02
16             3.166E+02
17             3.174E+02
18             3.180E+02
19             3.186E+02
20             3.187E+02
21             3.187E+02
22             3.188E+02
23             3.188E+02
24             3.189E+02
25             3.189E+02

```

element		heat flux	heat flux per unit area
number	center		
-----	-----	-----	-----
1	2.500E-03	-2.951E+00	-2.951E+04
2	7.500E-03	-2.966E+00	-2.966E+04
3	1.250E-02	-2.921E+00	-2.921E+04
4	1.750E-02	-2.941E+00	-2.941E+04
5	2.250E-02	-2.890E+00	-2.890E+04
6	2.750E-02	-2.882E+00	-2.882E+04
7	3.250E-02	-2.826E+00	-2.826E+04
8	3.750E-02	-2.697E+00	-2.697E+04
9	4.250E-02	-2.489E+00	-2.489E+04
10	4.750E-02	-2.229E+00	-2.229E+04
11	5.250E-02	-1.946E+00	-1.946E+04
12	5.750E-02	-1.669E+00	-1.669E+04
13	6.250E-02	-1.424E+00	-1.424E+04
14	6.750E-02	-1.208E+00	-1.208E+04
15	7.250E-02	-1.011E+00	-1.011E+04
16	7.750E-02	-8.336E-01	-8.336E+03
17	8.250E-02	-6.767E-01	-6.767E+03
18	8.750E-02	-5.391E-01	-5.391E+03
19	9.250E-02	-4.342E-01	-4.342E+03
20	9.750E-02	-3.519E-01	-3.519E+03
21	1.025E-01	-2.715E-01	-2.715E+03
22	1.075E-01	-1.929E-01	-1.929E+03
23	1.125E-01	-1.155E-01	-1.155E+03
24	1.175E-01	-3.827E-02	-3.827E+02

Results for time T = 8.000E+01:

node	temperature
number	-----
-----	-----
1	2.780E+02
2	2.783E+02
3	2.786E+02
4	2.790E+02
5	2.793E+02
6	2.796E+02
7	2.799E+02
8	2.858E+02
9	2.916E+02
10	2.970E+02
11	3.021E+02
12	3.067E+02
13	3.108E+02

14	3.122E+02
15	3.135E+02
16	3.146E+02
17	3.156E+02
18	3.163E+02
19	3.170E+02
20	3.171E+02
21	3.172E+02
22	3.173E+02
23	3.173E+02
24	3.174E+02
25	3.174E+02

element			
number	center	heat flux	heat flux per unit area
-----	-----	-----	-----
1	2.500E-03	-2.566E+00	-2.566E+04
2	7.500E-03	-2.566E+00	-2.566E+04
3	1.250E-02	-2.554E+00	-2.554E+04
4	1.750E-02	-2.551E+00	-2.551E+04
5	2.250E-02	-2.533E+00	-2.533E+04
6	2.750E-02	-2.522E+00	-2.522E+04
7	3.250E-02	-2.491E+00	-2.491E+04
8	3.750E-02	-2.414E+00	-2.414E+04
9	4.250E-02	-2.286E+00	-2.286E+04
10	4.750E-02	-2.120E+00	-2.120E+04
11	5.250E-02	-1.929E+00	-1.929E+04
12	5.750E-02	-1.726E+00	-1.726E+04
13	6.250E-02	-1.532E+00	-1.532E+04
14	6.750E-02	-1.349E+00	-1.349E+04
15	7.250E-02	-1.171E+00	-1.171E+04
16	7.750E-02	-1.001E+00	-1.001E+04
17	8.250E-02	-8.383E-01	-8.383E+03
18	8.750E-02	-6.837E-01	-6.837E+03
19	9.250E-02	-5.569E-01	-5.569E+03
20	9.750E-02	-4.542E-01	-4.542E+03
21	1.025E-01	-3.524E-01	-3.524E+03
22	1.075E-01	-2.513E-01	-2.513E+03
23	1.125E-01	-1.504E-01	-1.504E+03
24	1.175E-01	-5.024E-02	-5.024E+02

Results for time T = 1.000E+02:

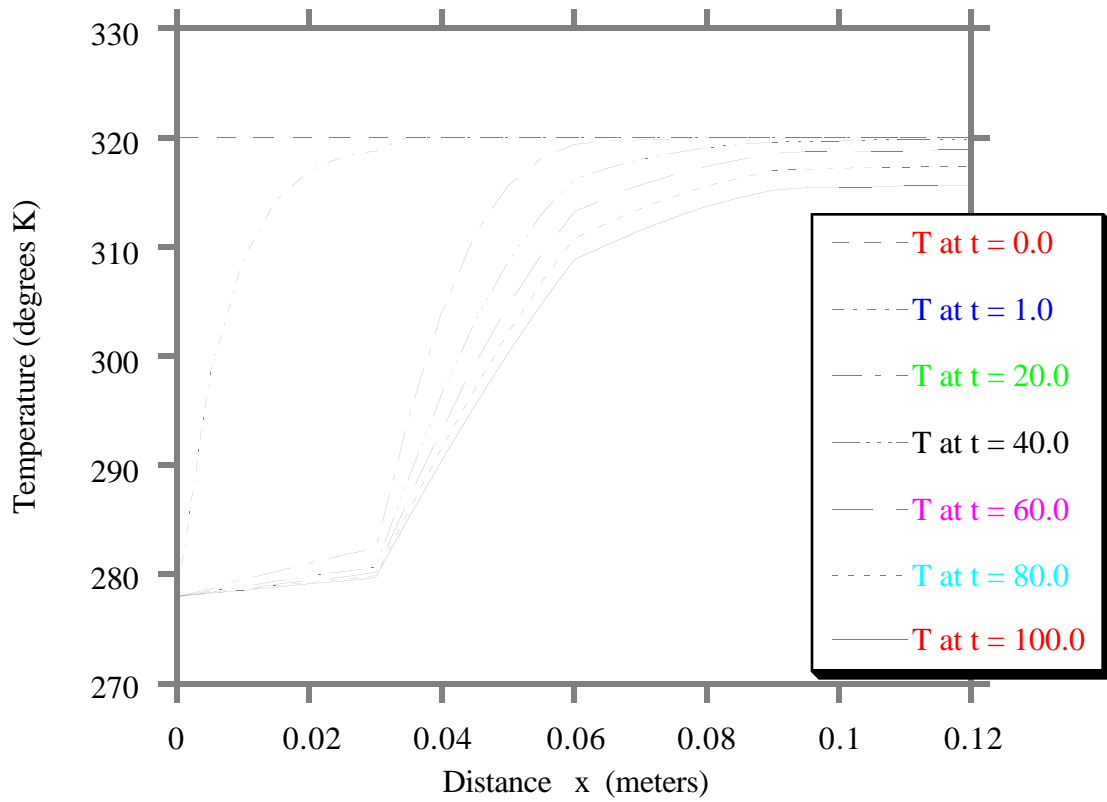
node	
number	temperature
-----	-----
1	2.780E+02

2	2.783E+02
3	2.786E+02
4	2.789E+02
5	2.792E+02
6	2.794E+02
7	2.797E+02
8	2.852E+02
9	2.904E+02
10	2.955E+02
11	3.003E+02
12	3.047E+02
13	3.088E+02
14	3.102E+02
15	3.115E+02
16	3.127E+02
17	3.137E+02
18	3.145E+02
19	3.152E+02
20	3.154E+02
21	3.155E+02
22	3.155E+02
23	3.156E+02
24	3.156E+02
25	3.157E+02

	element		
number	center	heat flux	heat flux per unit area
-----	-----	-----	-----
1	2.500E-03	-2.326E+00	-2.326E+04
2	7.500E-03	-2.324E+00	-2.324E+04
3	1.250E-02	-2.320E+00	-2.320E+04
4	1.750E-02	-2.315E+00	-2.315E+04
5	2.250E-02	-2.305E+00	-2.305E+04
6	2.750E-02	-2.295E+00	-2.295E+04
7	3.250E-02	-2.275E+00	-2.275E+04
8	3.750E-02	-2.222E+00	-2.222E+04
9	4.250E-02	-2.131E+00	-2.131E+04
10	4.750E-02	-2.010E+00	-2.010E+04
11	5.250E-02	-1.865E+00	-1.865E+04
12	5.750E-02	-1.702E+00	-1.702E+04
13	6.250E-02	-1.537E+00	-1.537E+04
14	6.750E-02	-1.376E+00	-1.376E+04
15	7.250E-02	-1.214E+00	-1.214E+04
16	7.750E-02	-1.053E+00	-1.053E+04
17	8.250E-02	-8.933E-01	-8.933E+03
18	8.750E-02	-7.352E-01	-7.352E+03
19	9.250E-02	-6.019E-01	-6.019E+03
20	9.750E-02	-4.919E-01	-4.919E+03
21	1.025E-01	-3.821E-01	-3.821E+03
22	1.075E-01	-2.732E-01	-2.732E+03
23	1.125E-01	-1.638E-01	-1.638E+03
24	1.175E-01	-5.455E-02	-5.455E+02

Graphically, these results are summarized in the following manner:

Results of the Sample Analysis : Temperature Time History



APPENDIX A

List of Significant Variables and Arrays Used in the Program

- variables controlling the maximum size of a problem:

MAXBND = maximum size of the half-bandwidth of the global conductivity matrix.

MAXELM = maximum number of elements permitted in an analysis.

MAXLOD = maximum number of records describing load set data.

MAXMAT = maximum number of material types.

MAXNOD = maximum number of node points permitted in an analysis.

MAXSPE = maximum number of records describing nodal specifications.

MAXSTP = maximum number of solution time steps to be printed.

- logical unit numbers (LUN's):

NUMIN = LUN associated with input.

NUMOUT = LUN associated with output.

- variables controlling the maximum size of a problem:

NUMELM = number of elements in the mesh being analyzed.

NUMLOD = number of records used to describe load sets associated with the present analysis.

NUMMAT = number of material associated with the present analysis.

NUMNOD = number of node points associated with the present analysis.

NUMSPE = number of records used to describe node point specifications.

- real variables:

THETA : parameter defining the particular time-marching algorithm used for the solution.

- integer arrays:

IFLAG : array defining the type of specification at a given node.

LODNUM : array storing the load set number associated with a given element.

KOUNT : keeps track of which nodes and/or elements have been considered in generation.

MATNUM : array storing the material number associated with a given element.

NOD : array containing the numbers of the two nodes associated with a given element.

NODSPE : array containing the number of the nodes at which specifications have been made.

- real arrays:

AREA : array containing the cross-sectional area of each element.

CONDUC : array containing the conductivity associated with a material type.

- DENSTY : array containing the mass density associated with a material type.
 ELEMC : element capacitance matrix.
 ELEMF : element right hand side vector.
 ELEMK : element coefficient matrix.
 GLFEFF : system (global) array containing elements of

$$\left([C] + (\theta - 1) \Delta t [K] \right) \left\{ \hat{T}^n \right\} + \Delta t \{F\}$$

- GLLHS : system (global) “left-hand side” array containing elements of

$$\left([C] + \theta \Delta t [K] \right)$$

- GLOBC : system (global) capacitance matrix.
 GLOBF : system (global) right hand side vector.
 GLOBK : system (global) coefficient matrix.
 GLRHS : system (global) “right-hand side” array containing elements of

$$\left([C] + (\theta - 1) \Delta t [K] \right)$$

- SOLVEC : solution vector (i.e., the vector containing values of the primary dependent variable).
 SOURC1 : array storing the value of a source term associated with a given load set.
 SOURC2 : array storing the value of a source term associated with a given load set.
 TNEW : vector storing the temperature at the current solution step.
 TOLD : vector storing the temperature at the previous solution step.
 VALUE : array storing the value of a quantity specified at a node.
 X : array containing the coordinates of the node points.